

Origami Mathematics in Education

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Tools and Mathematics

29 November 2016

Origami

- The Art of Folding



Origami

- The Art of Folding



<http://img.gawkerassets.com/img/17jp3vs9qkjb6jpg/original.jpg>

<http://res.artnet.com/news-upload/2014/05/origami-6.jpg>

<http://www.joostlangeveldorigami.nl/fotos/historyoforigami/bug.jpg>

<http://i.ytimg.com/vi/5nZtibCqFxx/hqdefault.jpg>

Origami

- The Art of Folding



http://illusion.scene360.com/wp-content/themes/sahara-10/submissions/2012/10/jun_mitani_03.jpg

http://farm5.static.flickr.com/4107/4946857347_a17b3e7900.jpg

<http://giangdinh.com/wp-content/uploads/2013/09/prayer.jpg>

Origami

- The Art of Folding



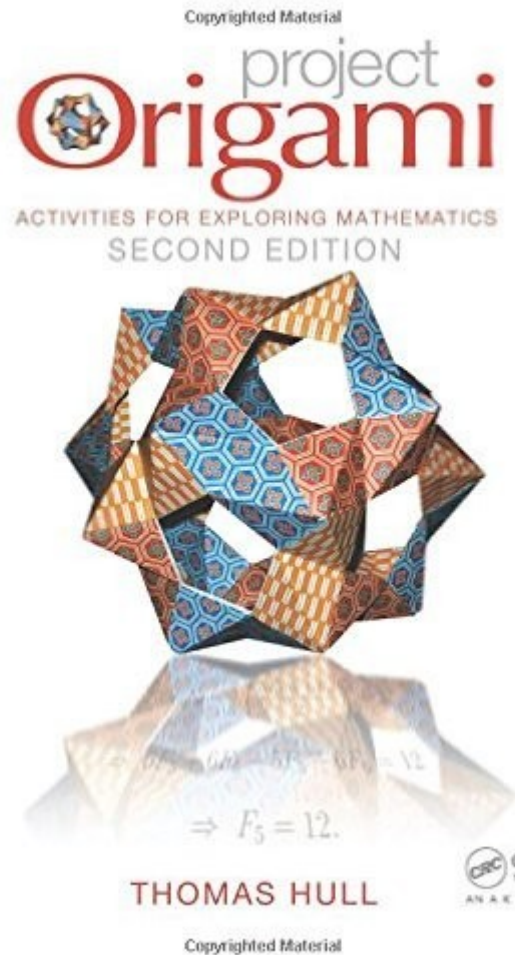
https://c2.staticflickr.com/4/3530/5835802683_a7ca138ff9.jpg

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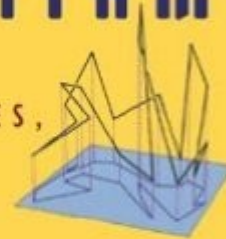
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Origami in the Classroom

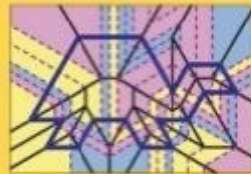


GEOMETRIC FOLDING ALGORITHMS

LINKAGES,



ORIGAMI,



POLYHEDRA

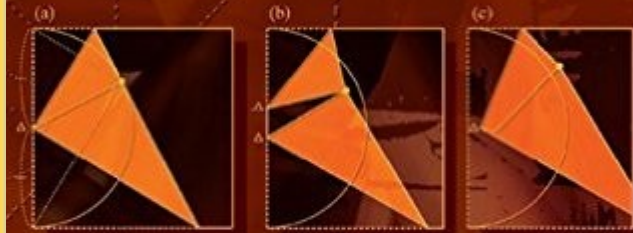


ERIK D. DEMAIN & JOSEPH O'ROURKE

ORIGAMICS

Mathematical Explorations through Paper Folding

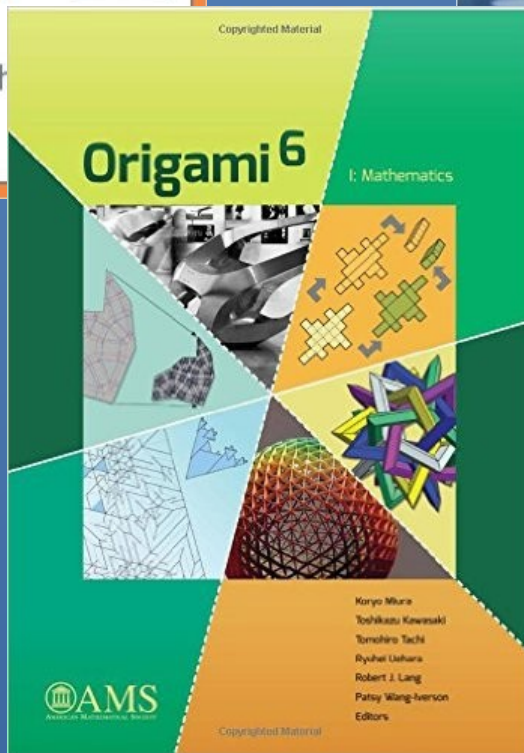
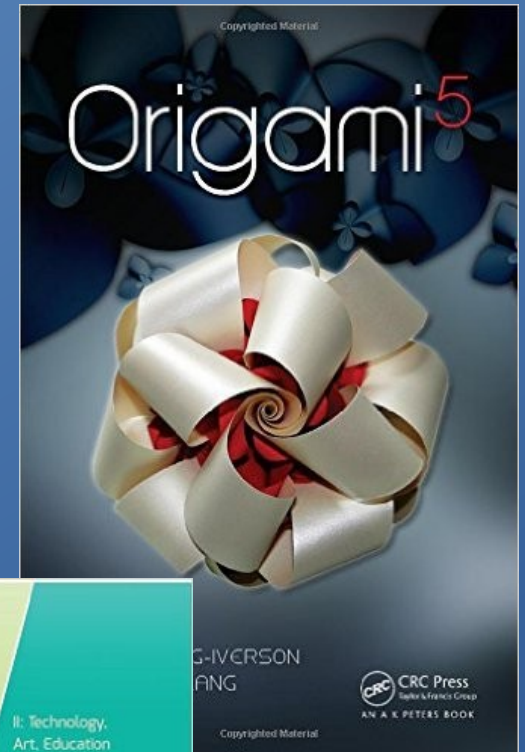
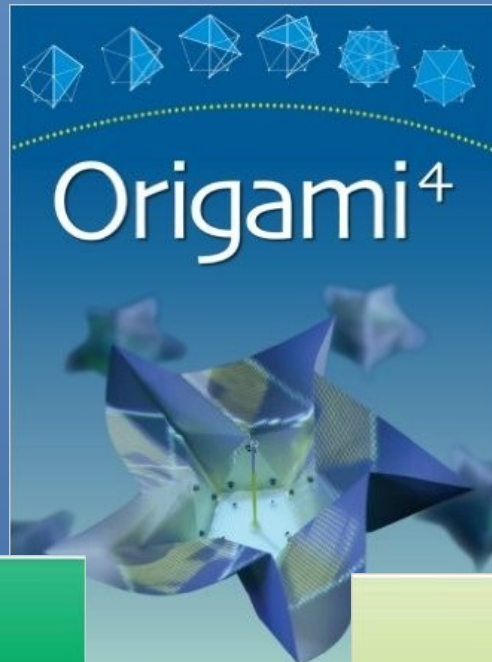
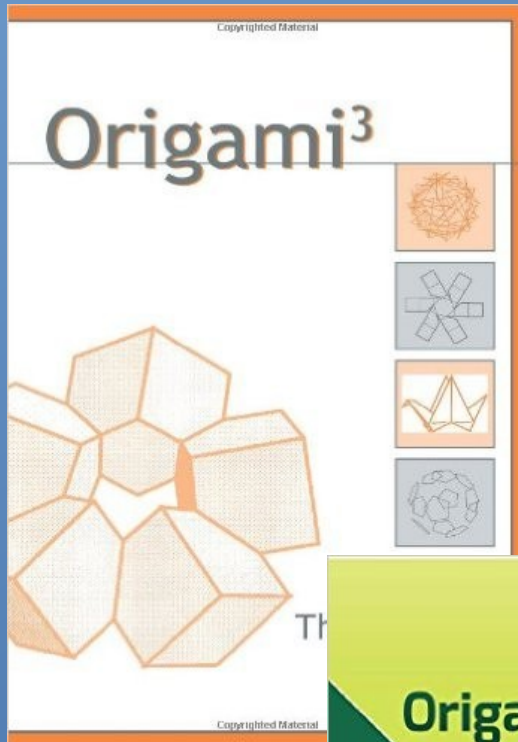
Kazuo Haga



edited and translated by
Josefina C. Fonacier
Masami Iwata

World Scientific

Origami Resources



1D Origami

Folding In Half

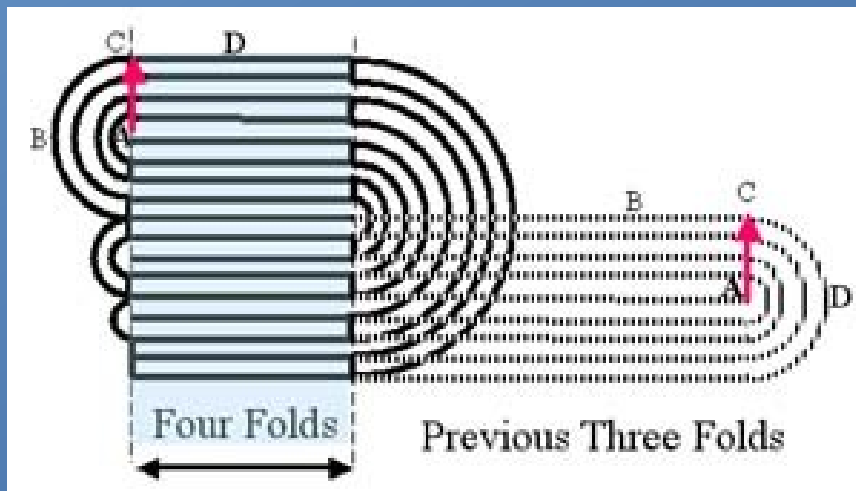
- How many times can you fold paper in half?
 - 8 times?

Folding In Half

- How many times can you fold paper in half?
 - 8 times?
- Is there an upper limit?

Folding In Half

- Britney Gallivan 2001



$$L = \frac{\pi \cdot t}{6} \cdot (2^n + 4)(2^n - 1)$$

$$W = \pi t 2^{3(n-1)/2}$$



Activity 1

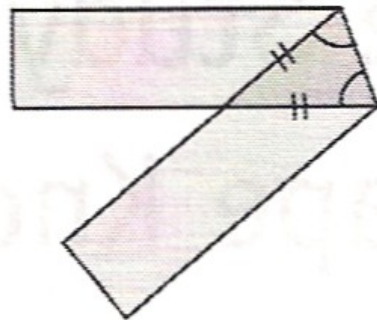
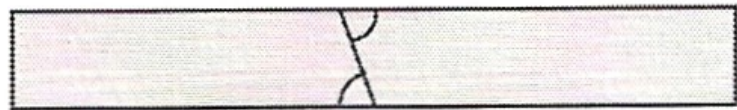
Parabolas

- Why does it work?
- Can other conics be constructed?
- What if you use non-flat paper?
- What can we learn concerning:
 - Parabolas ?
 - Envelopes?
 - Derivatives?
 - Tangents?
 - Convergence of sequences?

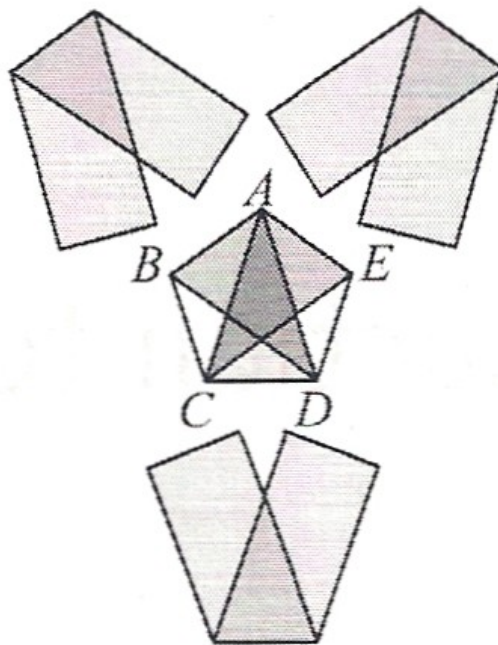
Knots



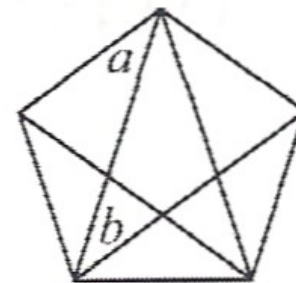
Knots



(a)



(b)



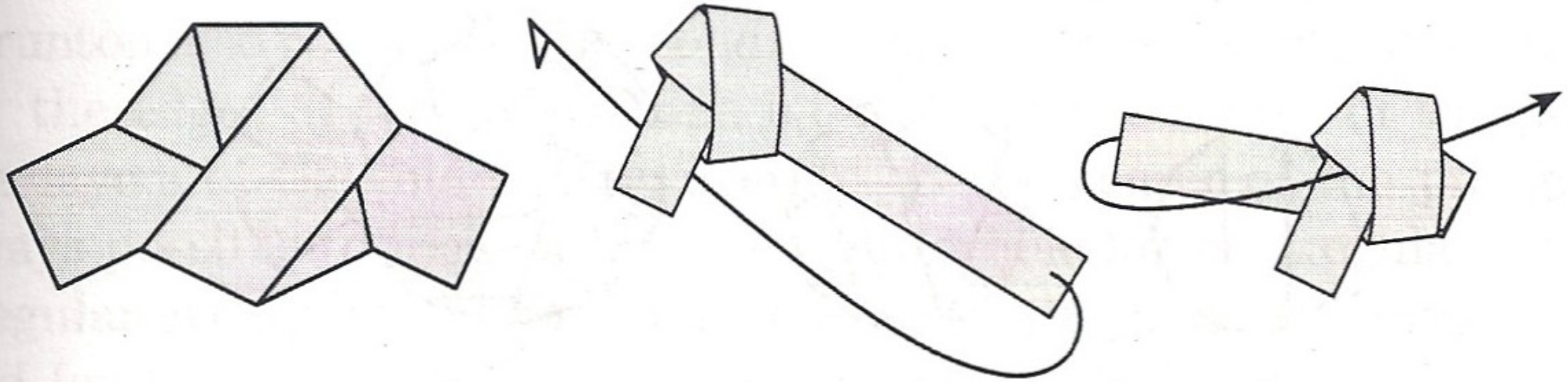
(c)

$$a=b$$

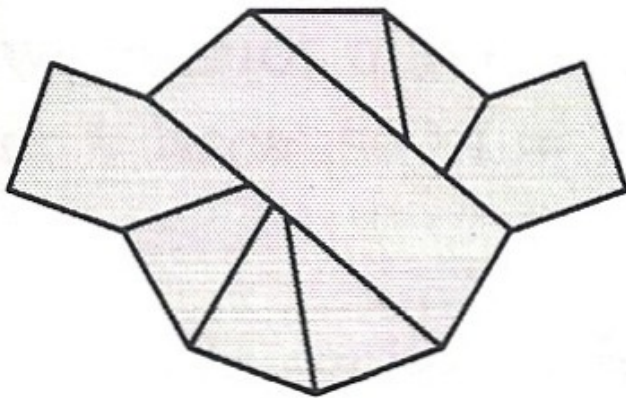


(d)

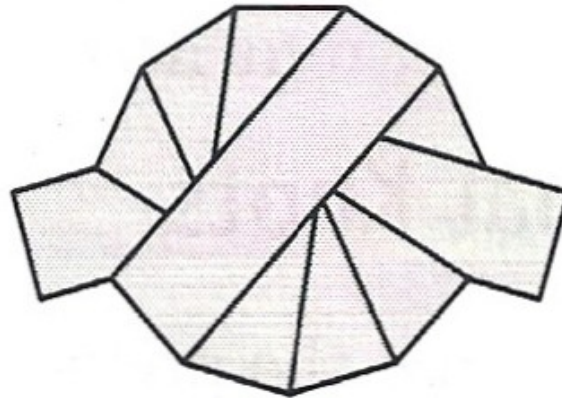
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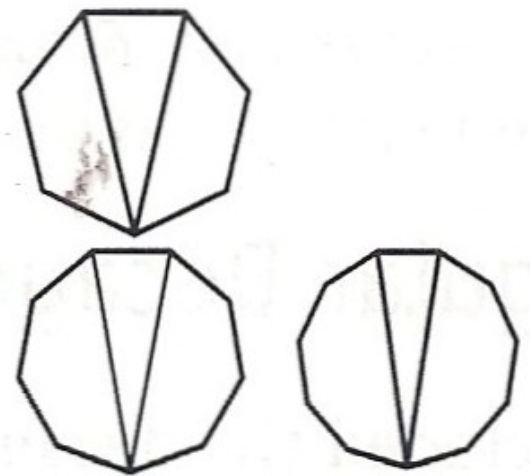
(a)



(b)

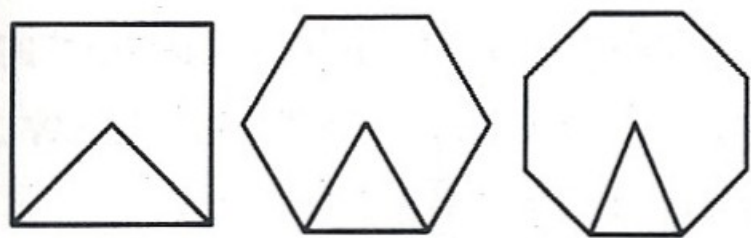


(c)

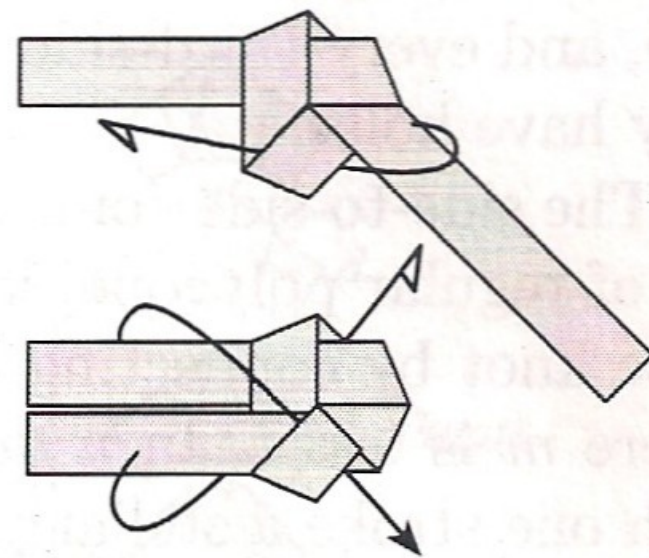
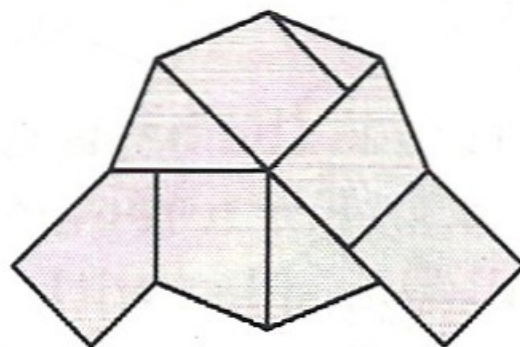


(d)

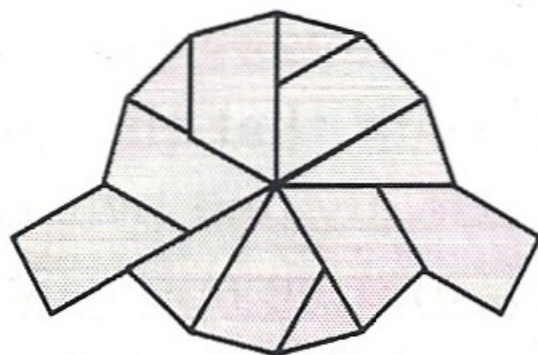
Knots



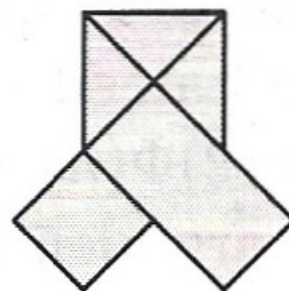
(a)



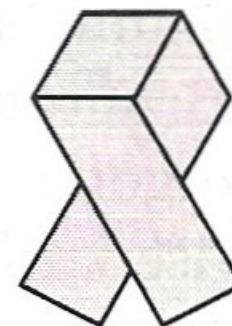
(b)



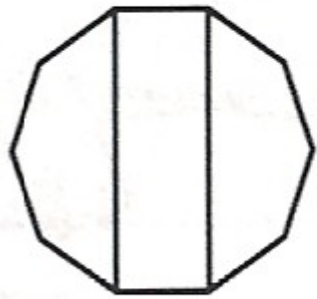
(c)



(d)



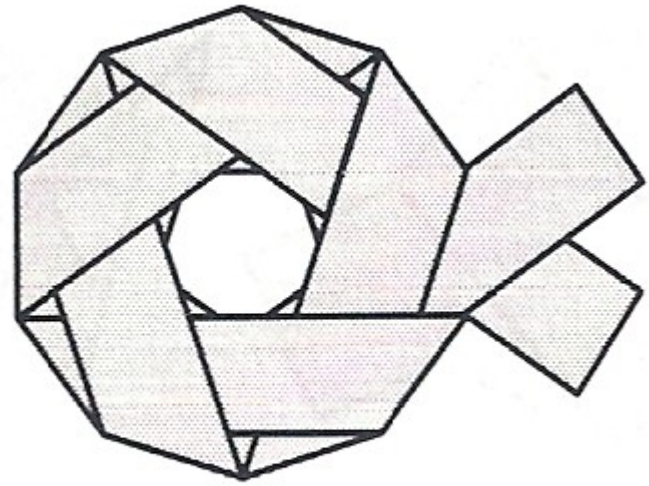
Knots



(a)

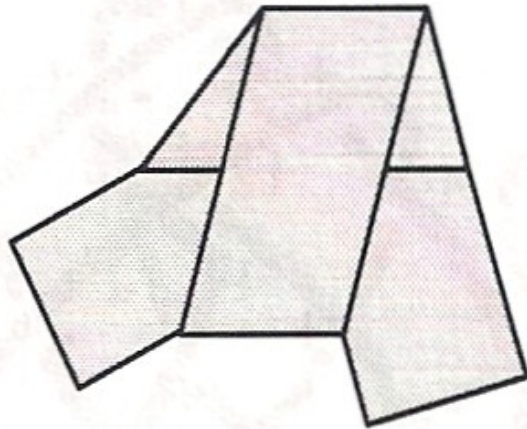
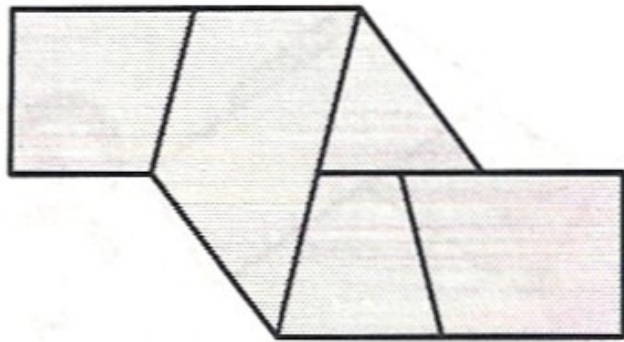


(b)

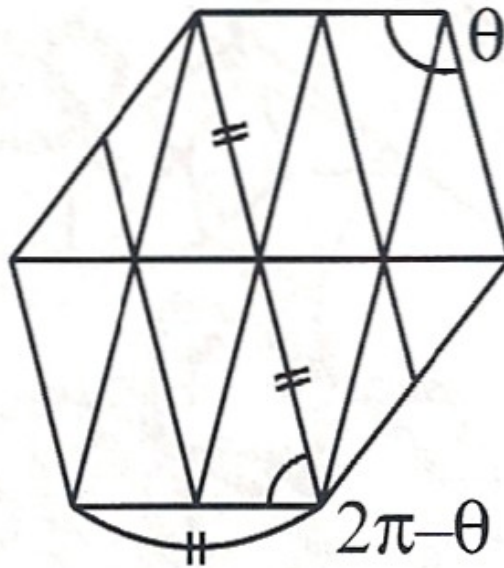


(c)

Knots

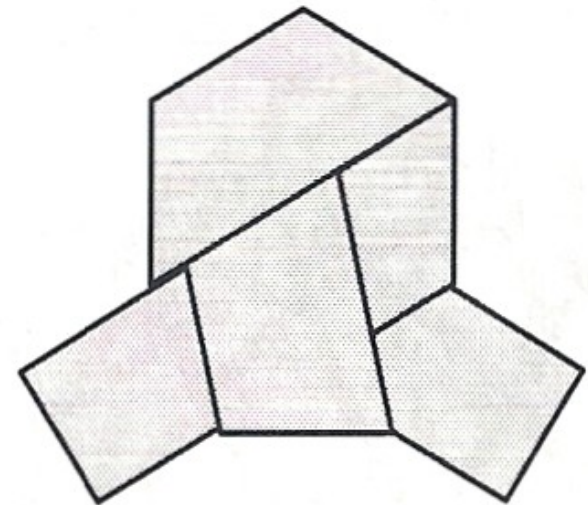


(a)



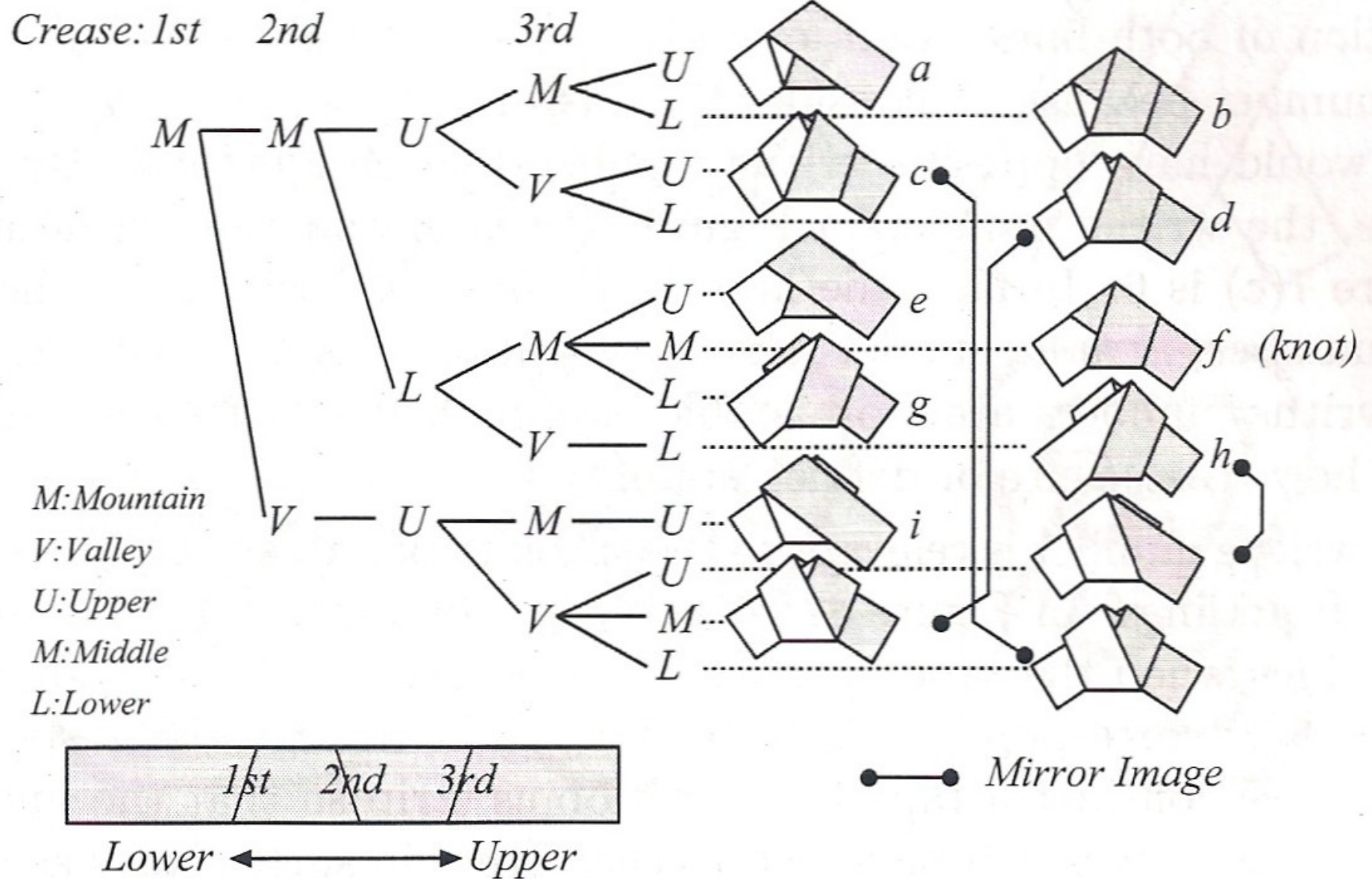
$$2\cos(2\pi - \theta) = 1/2$$

(b)

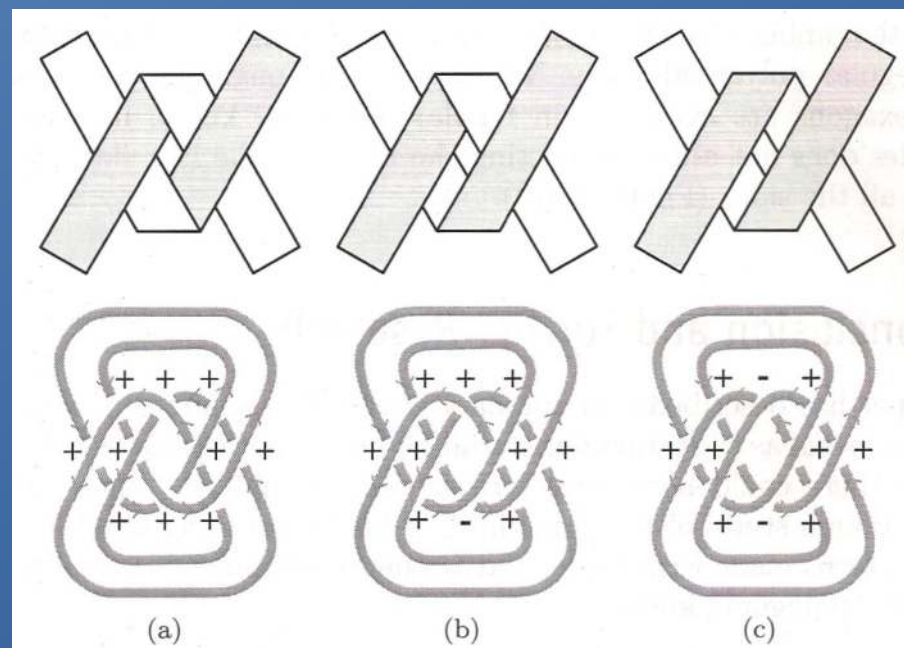
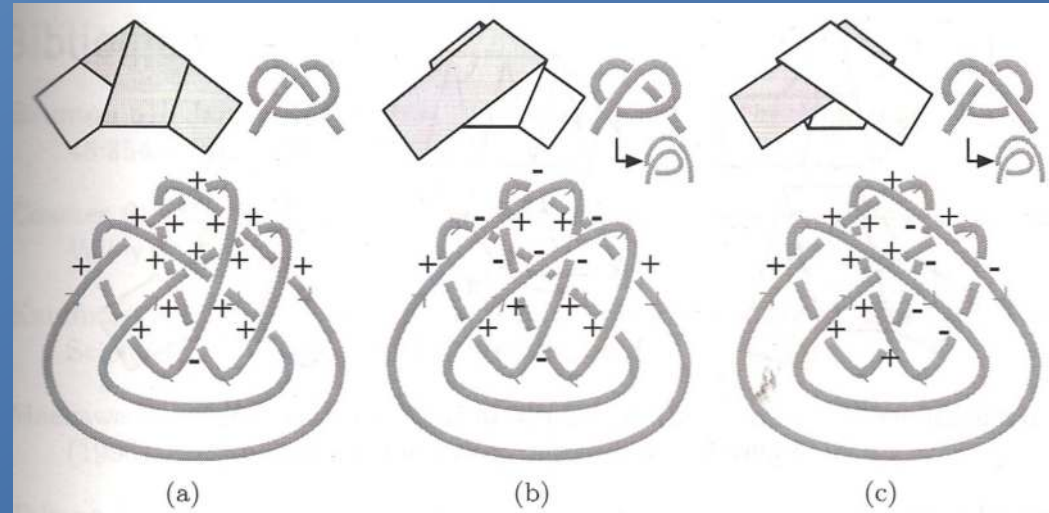
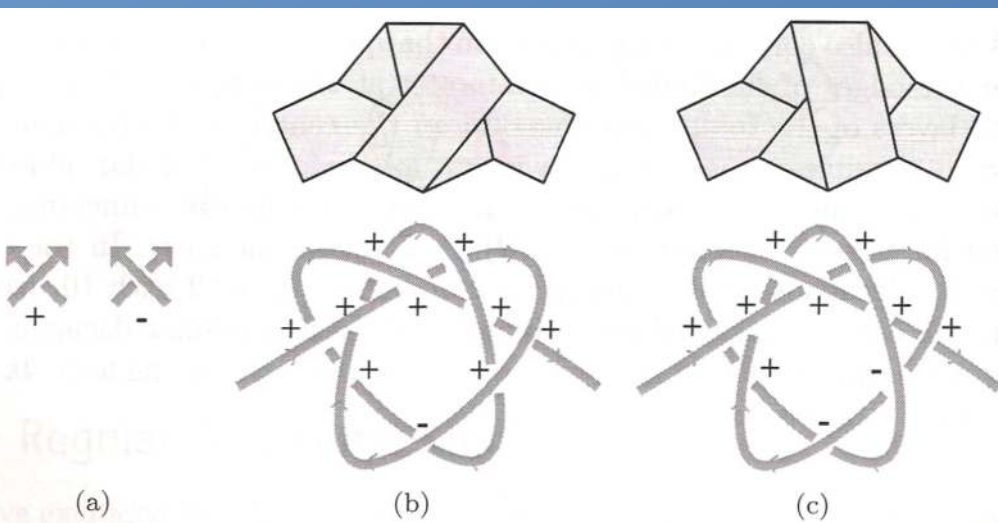


(c)

Knots



Knots



Knots

- Explorations:
 - Perimeter, area
 - Irregular patterns
 - Enumerations
 - Knot theory, topology

Activity 2

- Fujimoto approximation

Fujimoto Approximation

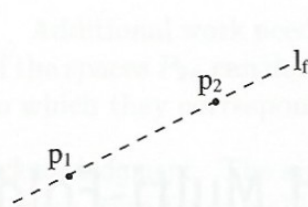
- Error is halved at each operation
- Repeating left-right pattern represented as the binary expansion of $1/n$
 - $1/5$: .00110011...
 - $1/7$: .011011011...

Between 1D and 2D

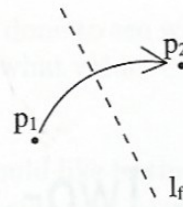
Origami Constructions

- What geometric constructions are possible?

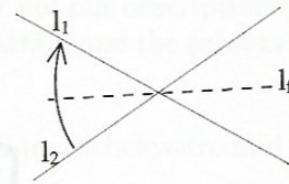
Origami Constructions



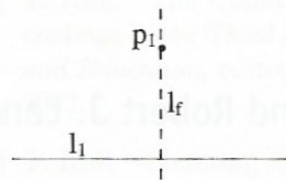
(O1) Given two points p_1 and p_2 , we can fold a line connecting them.



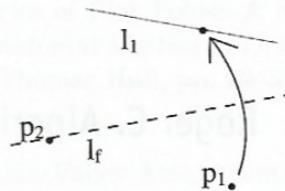
(O2) Given two points p_1 and p_2 , we can fold p_1 onto p_2 .



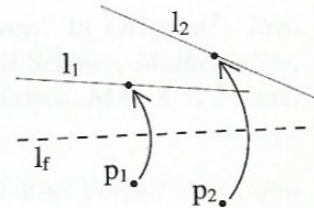
(O3) Given two lines l_1 and l_2 , we can fold line l_1 onto l_2 .



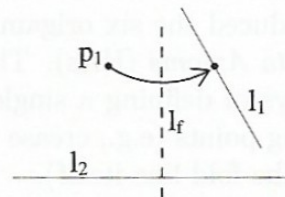
(O4) Given a point p_1 and a line l_1 , we can make a fold perpendicular to l_1 passing through the point p_1 .



(O5) Given two points p_1 and p_2 and a line l_1 , we can make a fold that places p_1 onto l_1 and passes through the point p_2 .

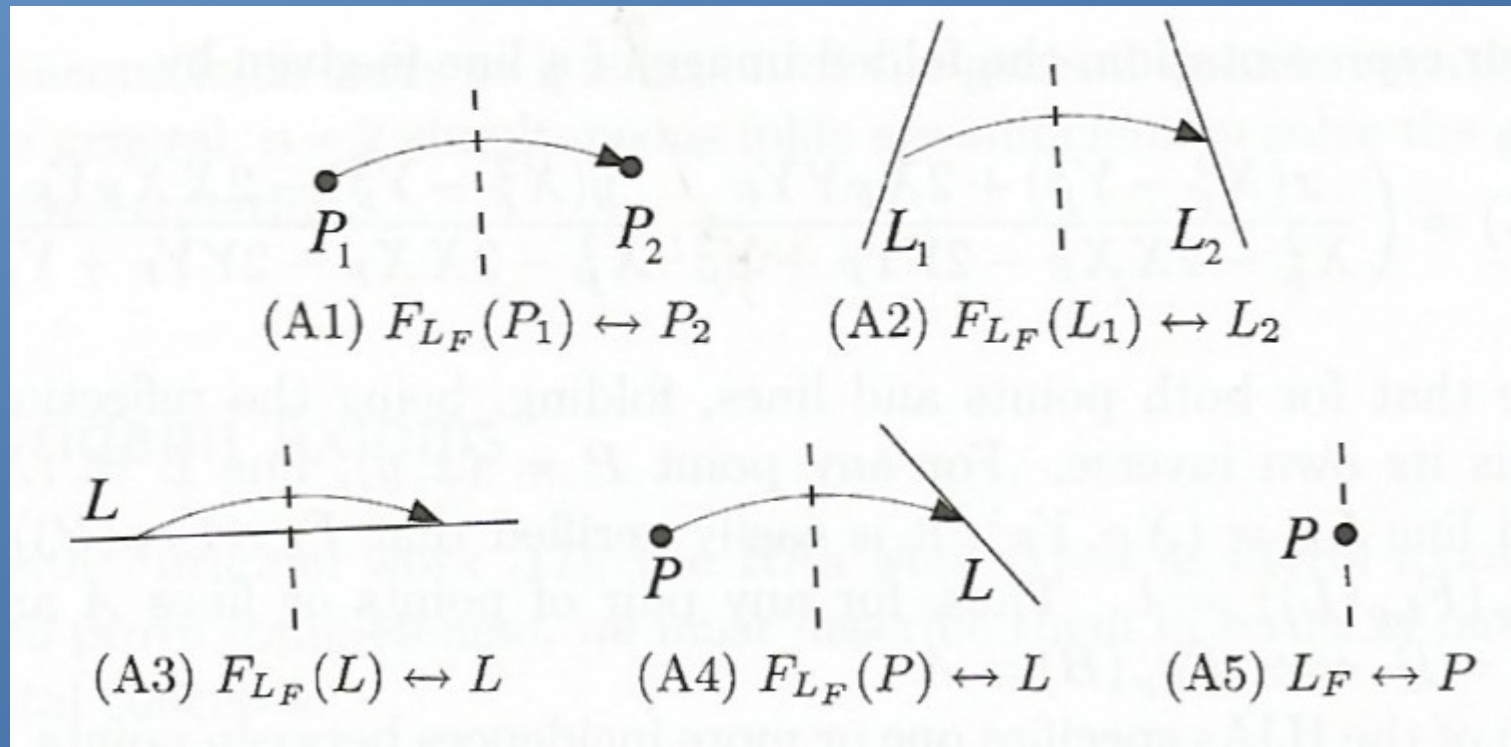


(O6) Given two points p_1 and p_2 and two lines l_1 and l_2 , we can make a fold that places p_1 onto line l_1 and places p_2 onto line l_2 .



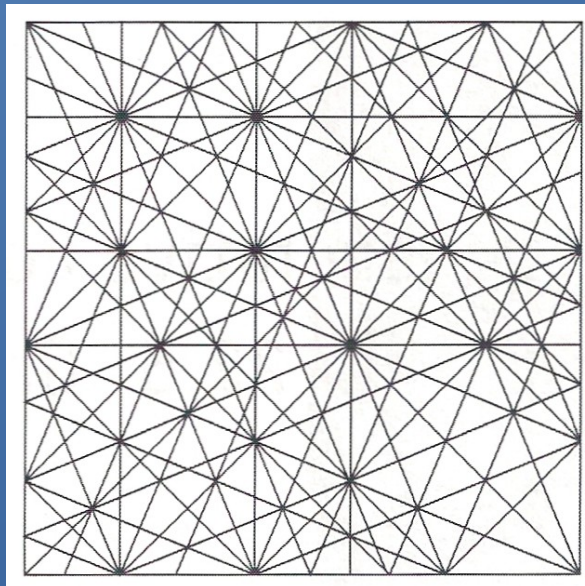
(O7) Given a point p_1 and two lines l_1 and l_2 , we can make a fold perpendicular to l_2 that places p_1 onto line l_1 .

Origami Constructions



Origami Constructions

- 22.5 degree angle restriction
 - All coordinates of the form $\frac{m+n\sqrt{2}}{2^l}$ are constructible
 - Algorithm linear in $l, \log(m), \log(n)$



Origami Constructions

- More generally:
 - Constructible numbers of the form $2^m 3^n$
 - Angle trisection, cube doubling possible
 - Roots of the general cubic

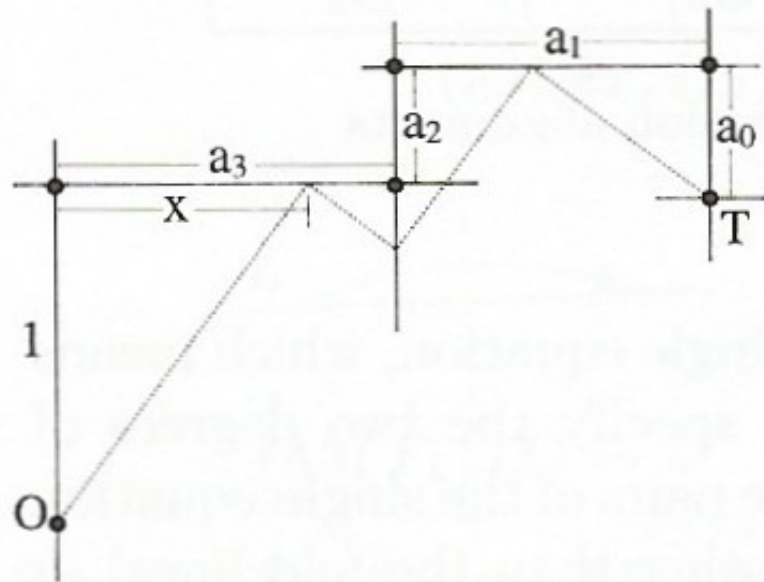
Origami Constructions

- Polynomial root finding, Lill's method

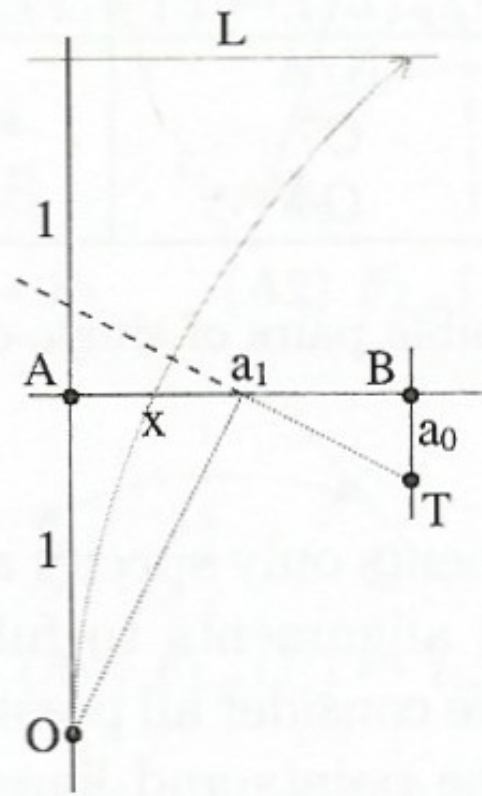
$$x^4 - a_3x^3 + a_2x^2 - a_1x - a_0 = 0$$

$$x^2 - a_1x - a_0 = 0$$

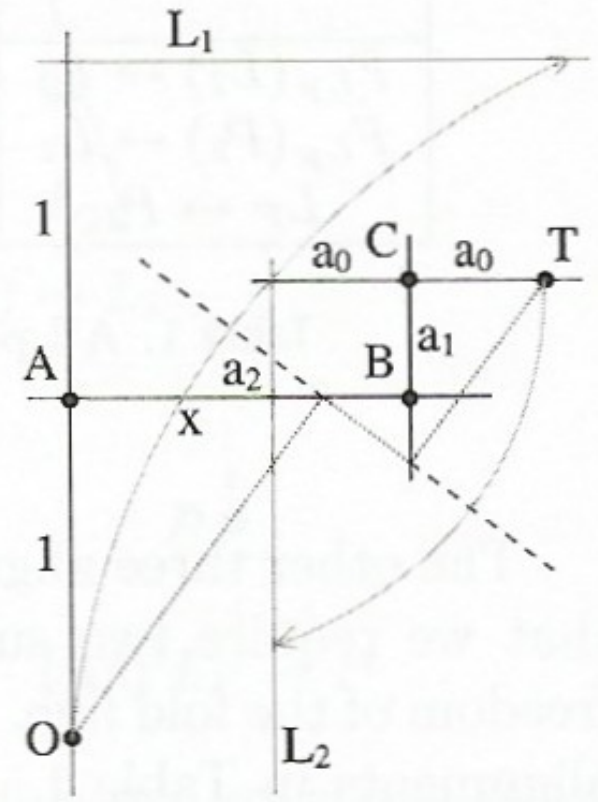
$$x^3 - a_2x^2 + a_1x - a_0 = 0$$



(a)



(b)

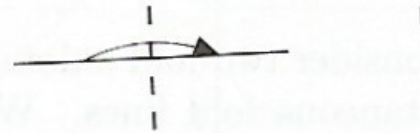


(c)

Origami Constructions



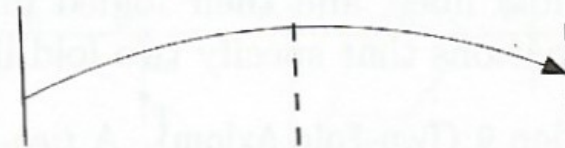
(AL1) $F_{L_{F_a}}(L_{F_b}) \leftrightarrow L_{F_b}$



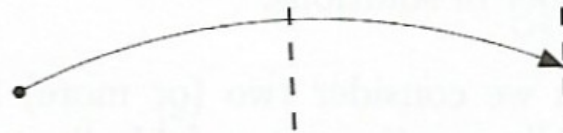
(AL2) $F_{L_{F_a}}(L) \leftrightarrow L$



(AL3) $L_{F_a} \leftrightarrow P$



(AL4) $F_{L_{F_a}}(L) \leftrightarrow L_{F_b}$



(AL5) $F_{L_{F_a}}(P) \leftrightarrow L_{F_b}$



(AL6) $F_{L_{F_a}}(P) \leftrightarrow L$



(AL7) $F_{L_{F_a}}(P) \leftrightarrow F_{L_{F_b}}(L)$



(AL8) $F_{L_{F_a}}(P_1) \leftrightarrow F_{L_{F_b}}(P_2)$



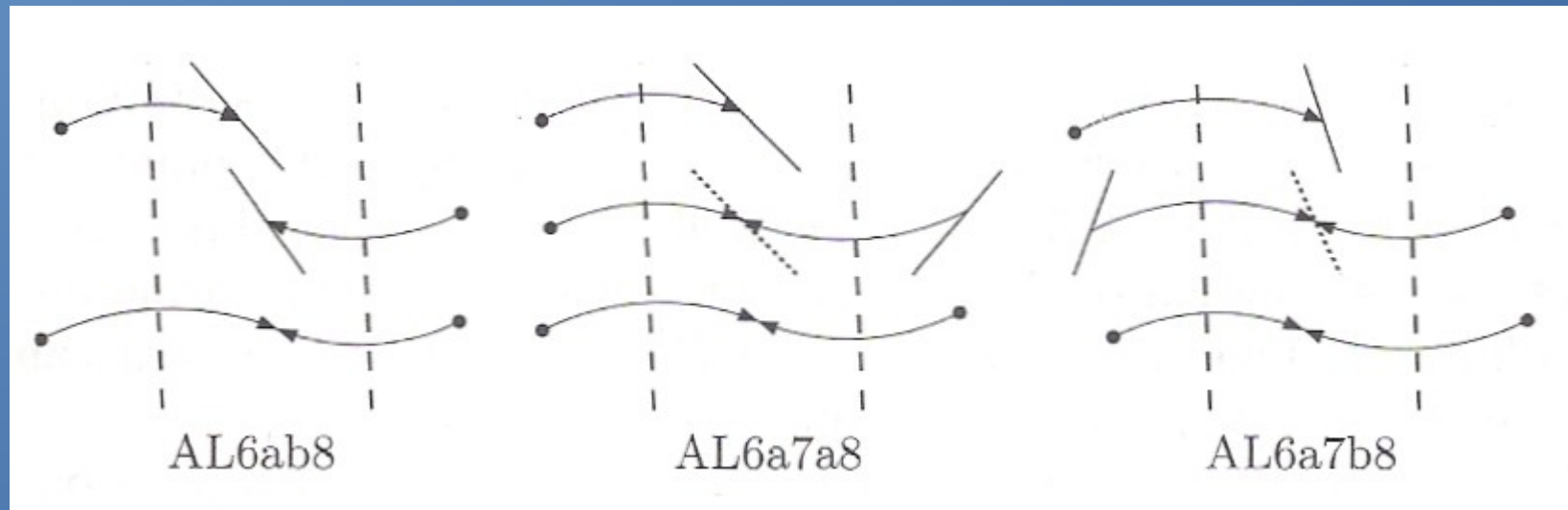
(AL9) $F_{L_{F_a}}(L_1) \leftrightarrow F_{L_{F_b}}(L_2)$



(AL10) $F_{L_{F_b}}(P_{L_{F_a}, L_1}) \leftrightarrow L_2$

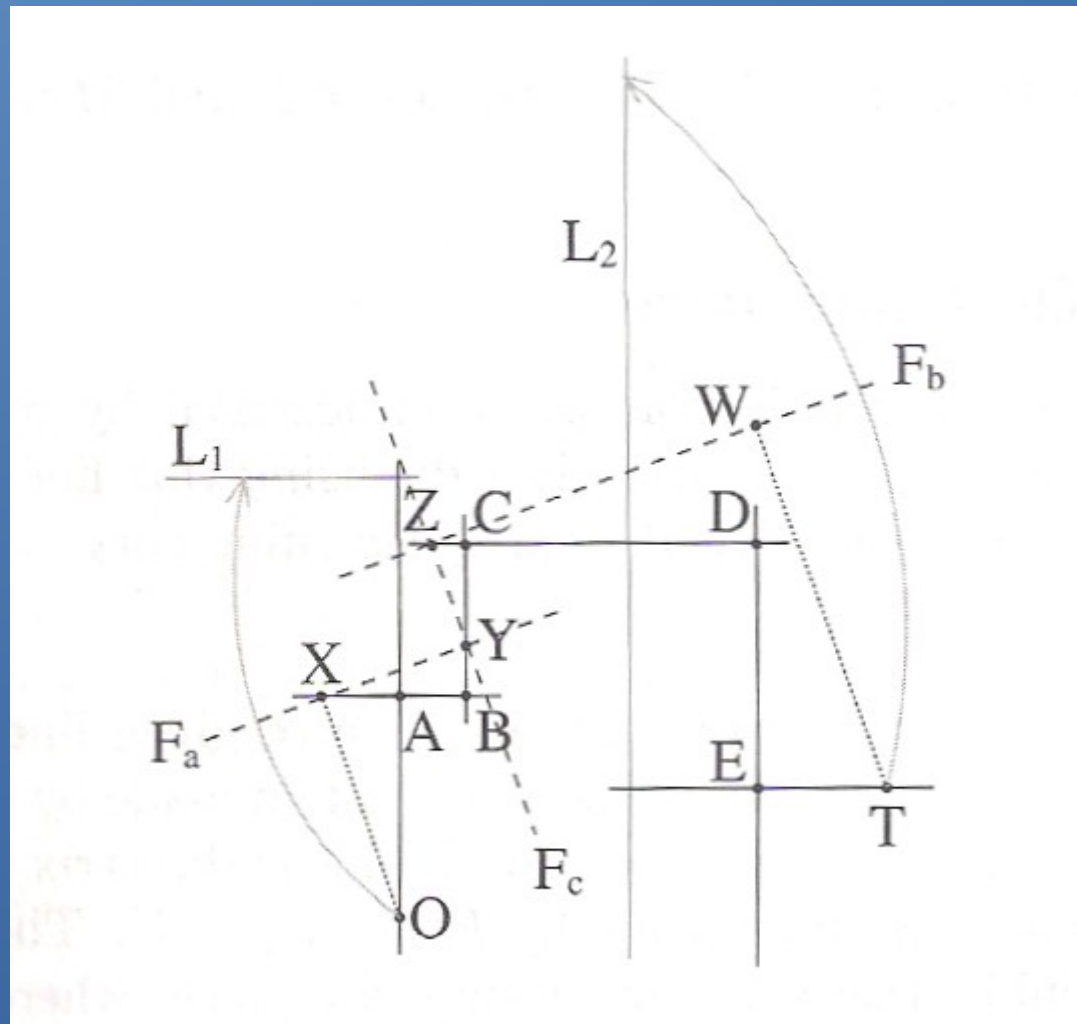
Origami Constructions

- 489 distinct two-fold line constructions



Origami Constructions

- General quintic construction



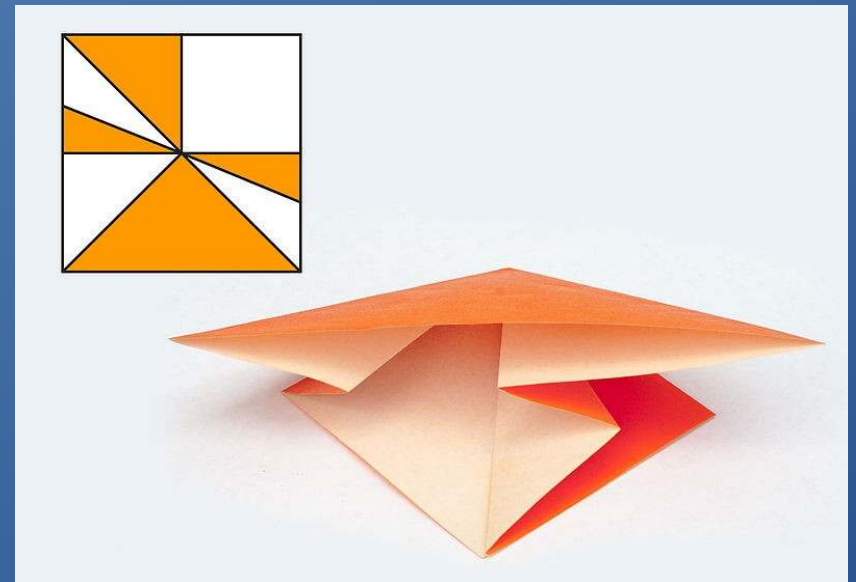
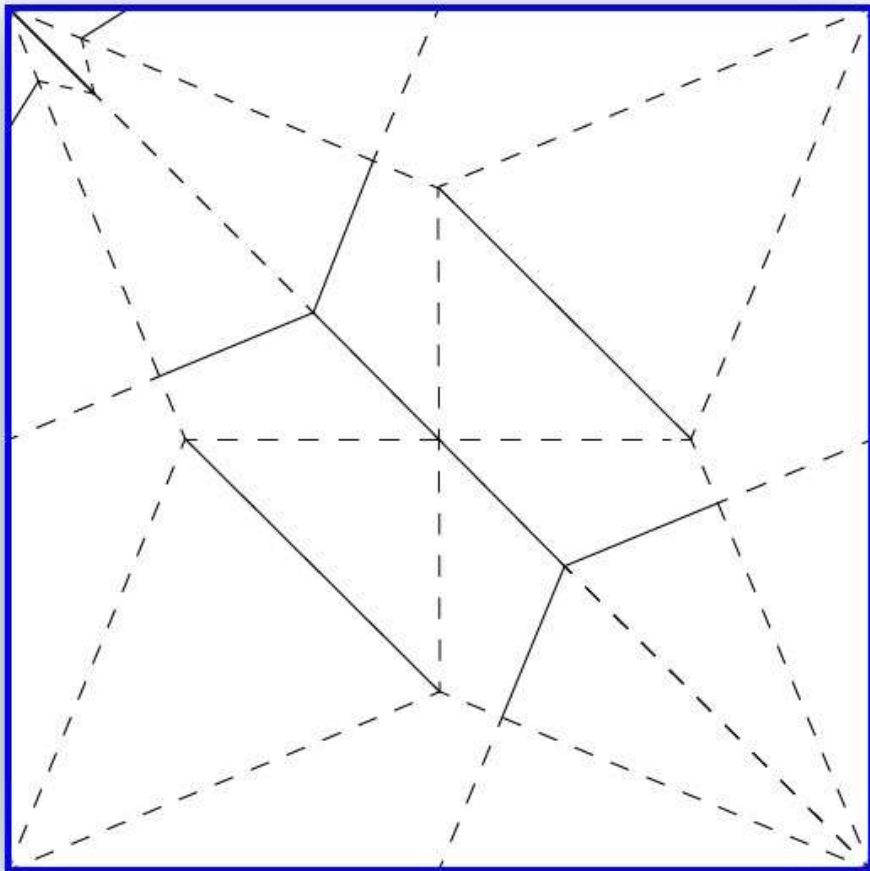
Origami Constructions

- Higher order equations, real solutions
 - Order n requires $(n-2)$ simultaneous folds
- What can we learn concerning:
 - Polynomial roots
 - Geometric constructions
 - Field theory
 - Galois theory

2D Folding

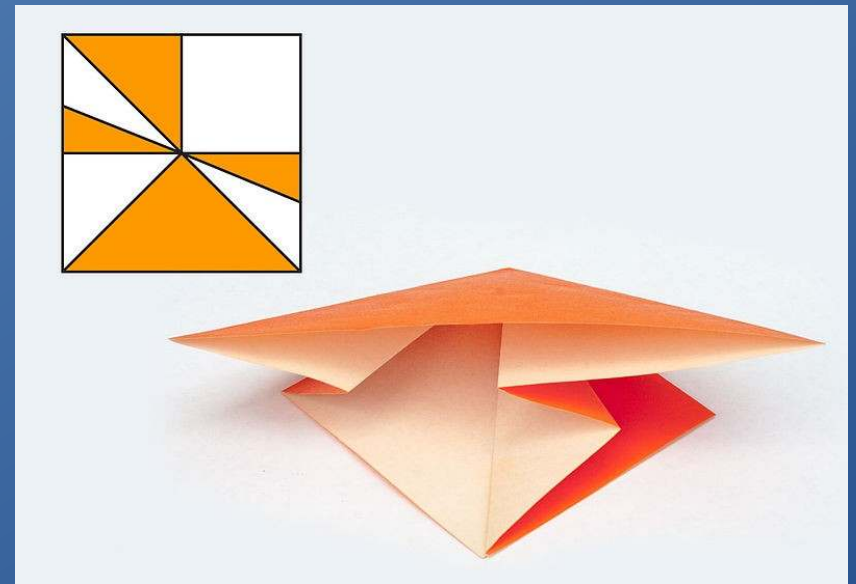
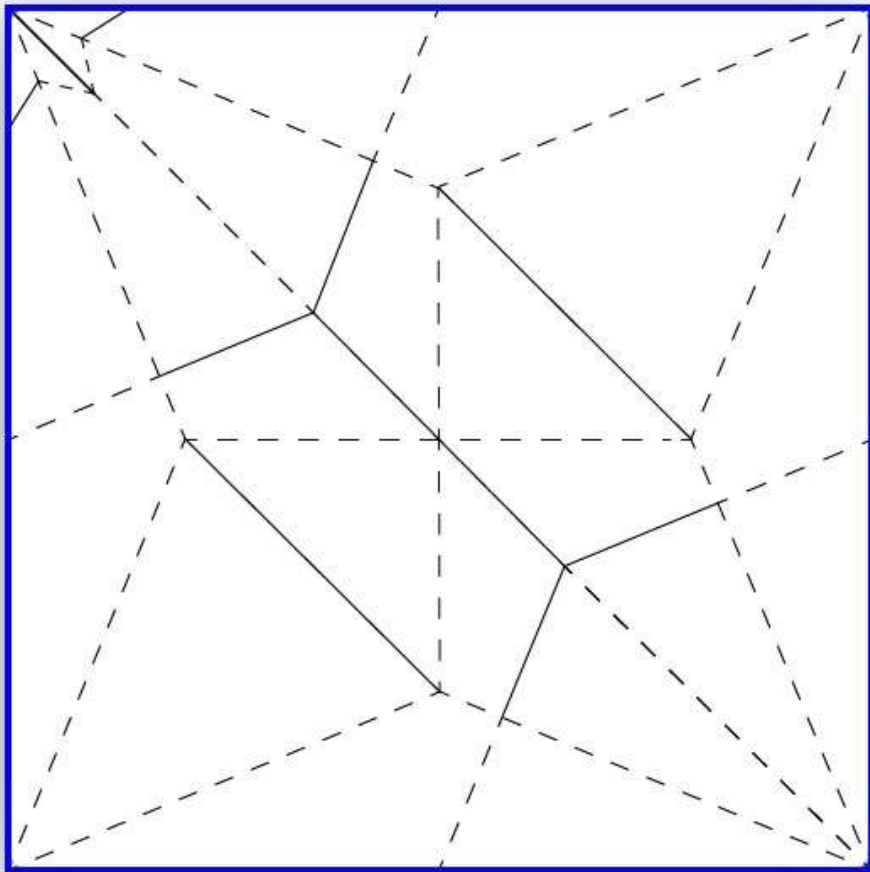
Flat Foldability Theorems

- Maekawa's theorem: $|M-V|=2$, even degrees



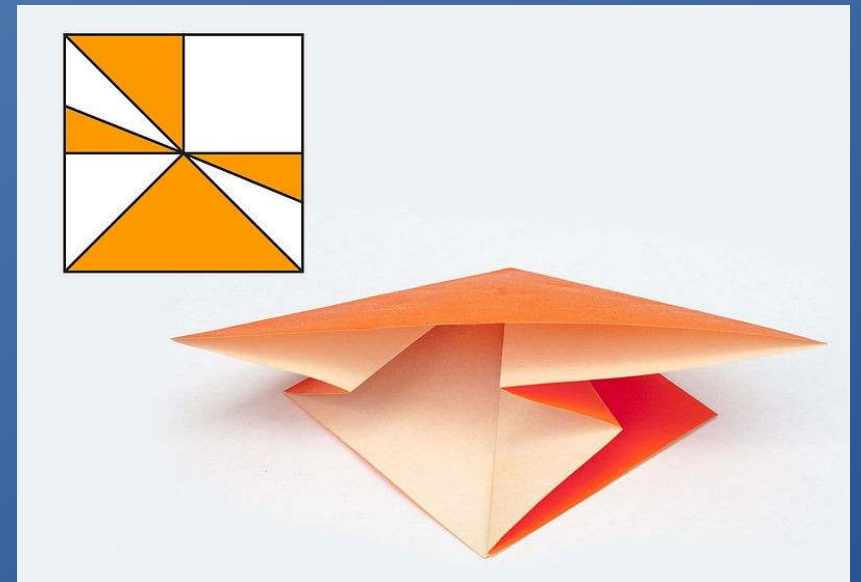
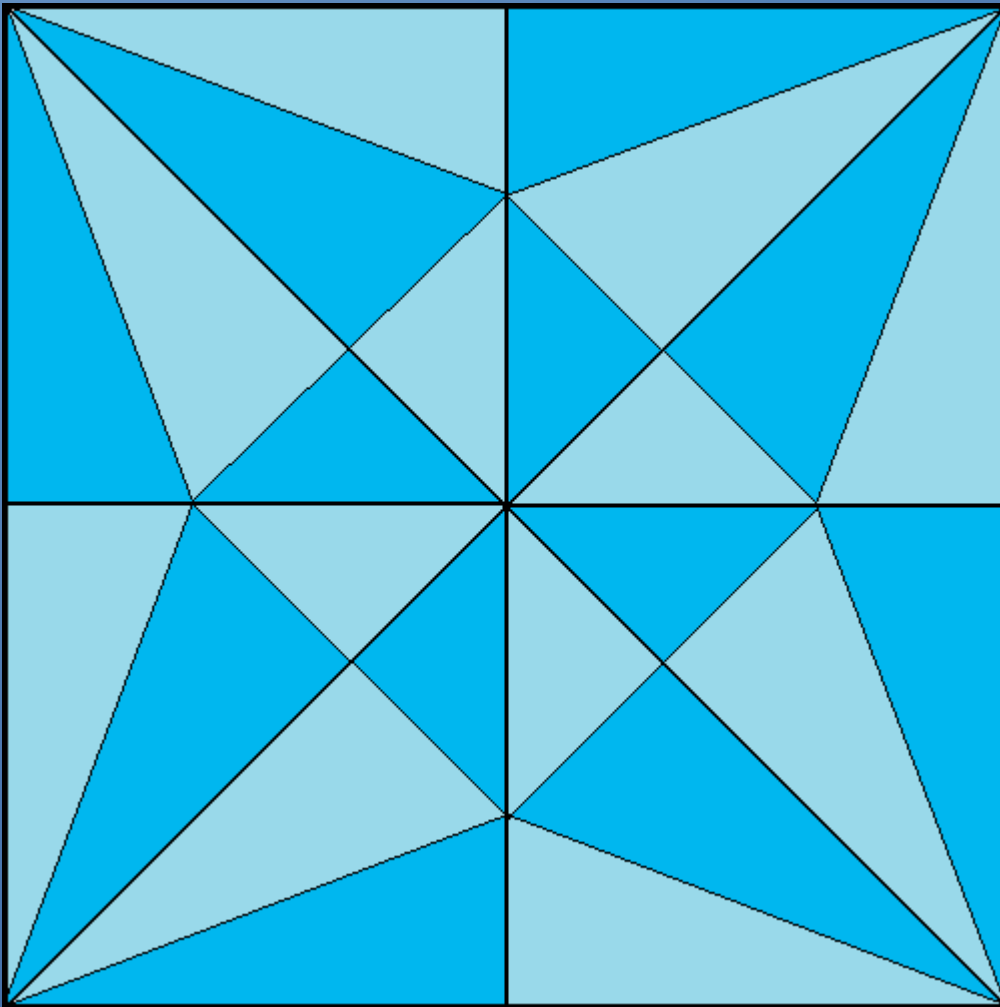
Flat Foldability Theorems

- Kawasaki's theorem: sum of alternating angles equals 180°



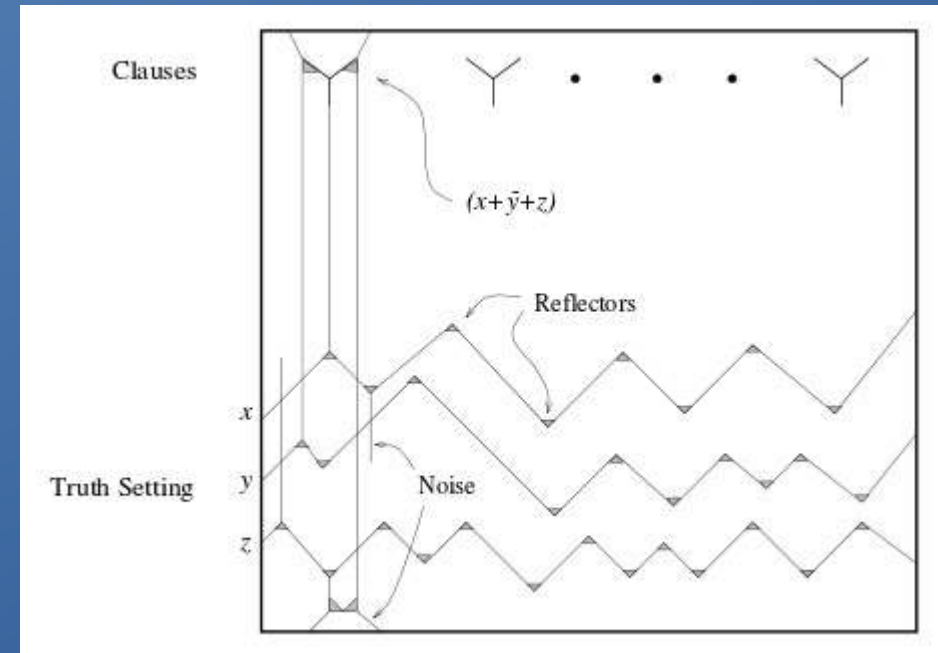
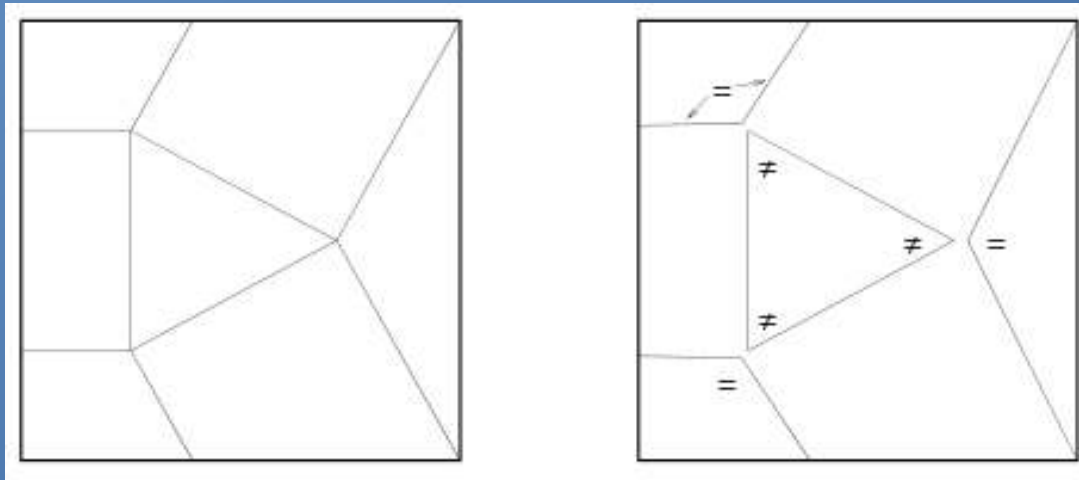
Flat Foldability Theorems

- Crease patterns are two-colorable



Flat Foldability is Hard

- Deciding flat-foldability is NP-complete



Circle Packing

- What is a flap?

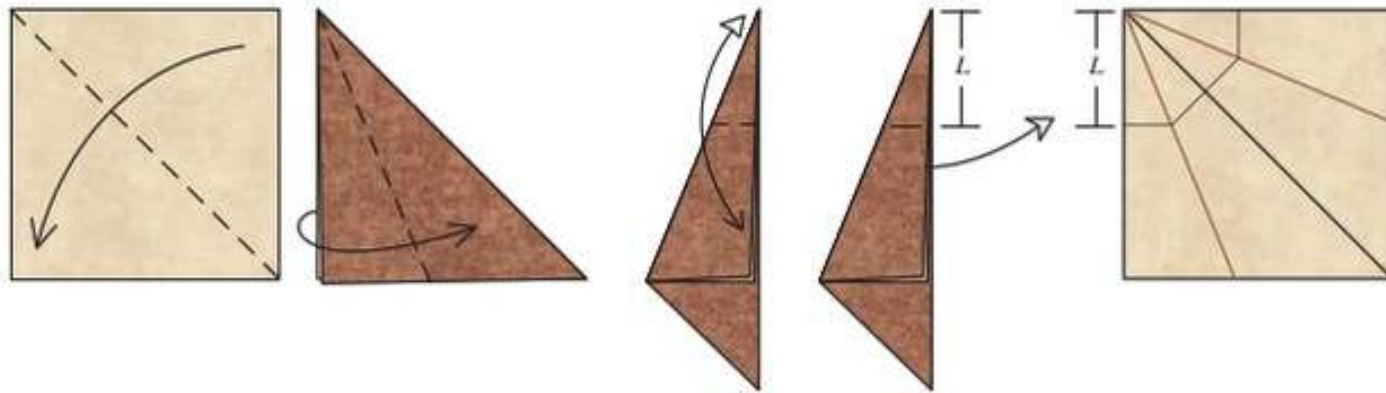
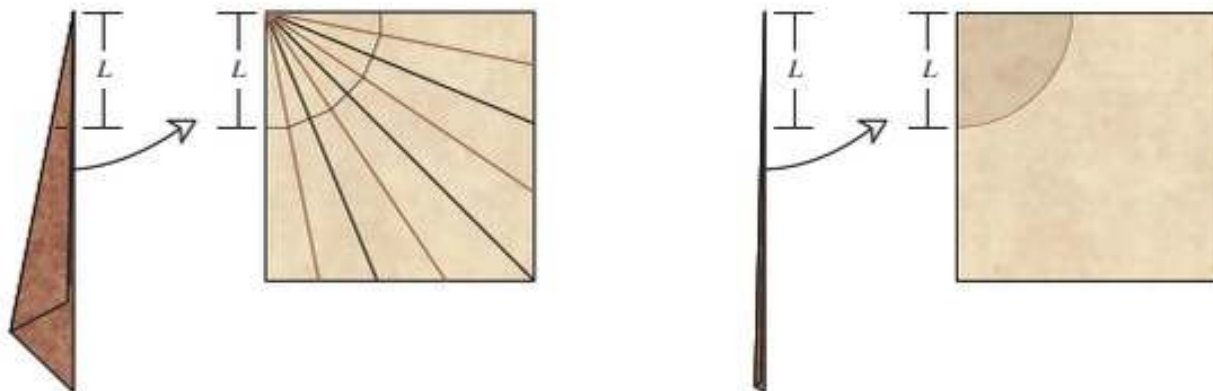


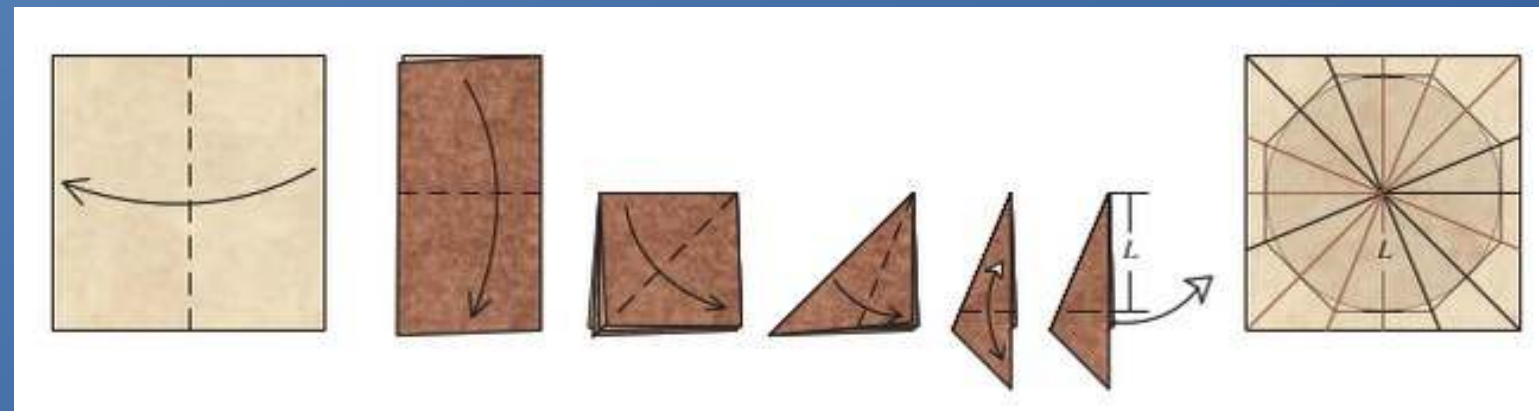
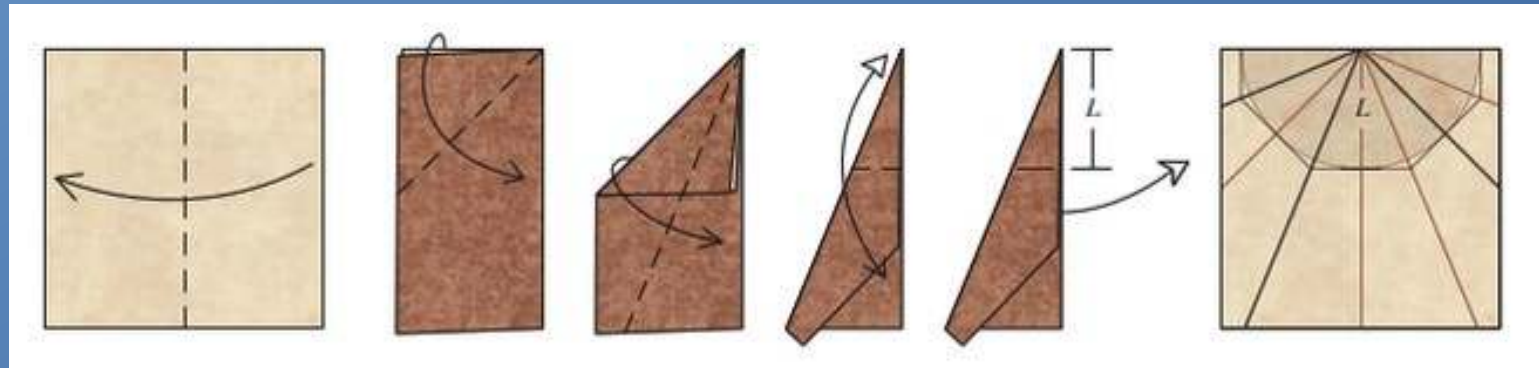
Figure 9.2.

Folding a corner flap of length L from a square.



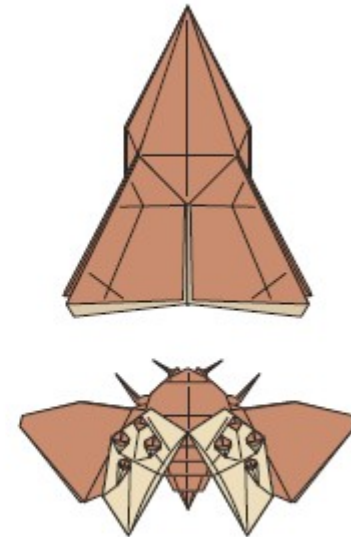
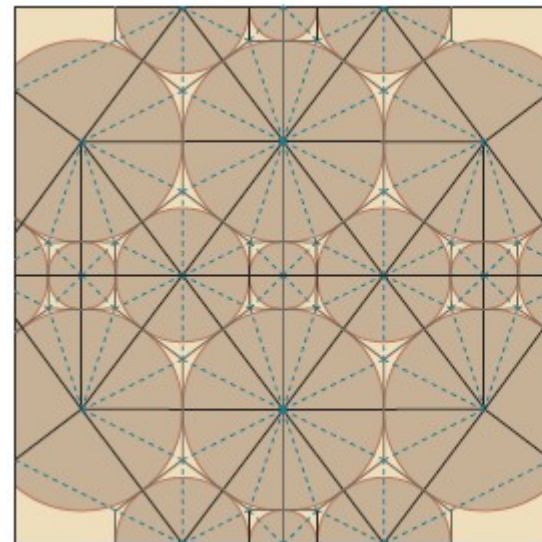
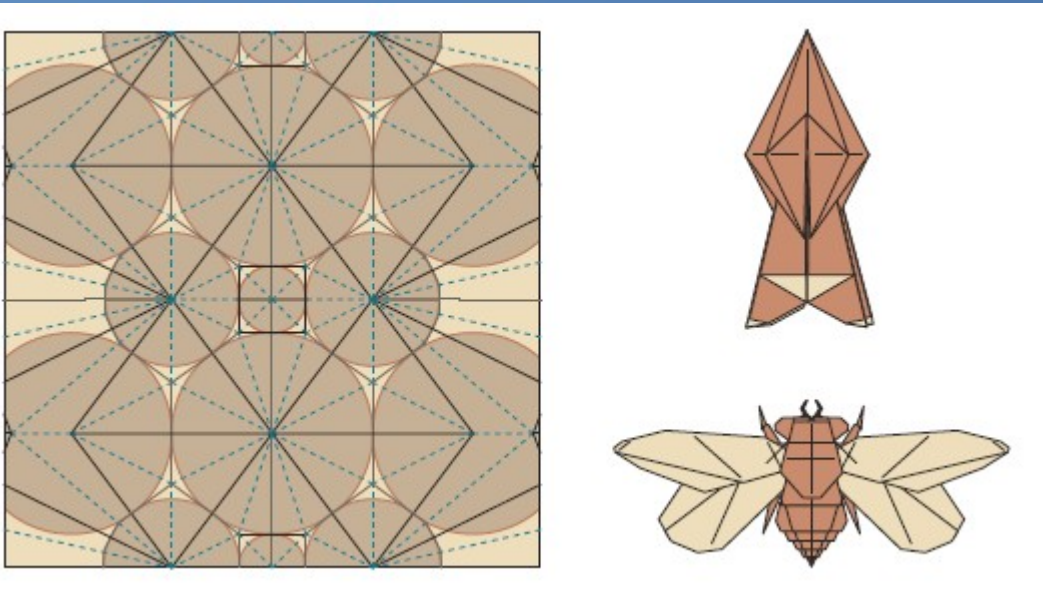
Circle Packing

- What is a flap?



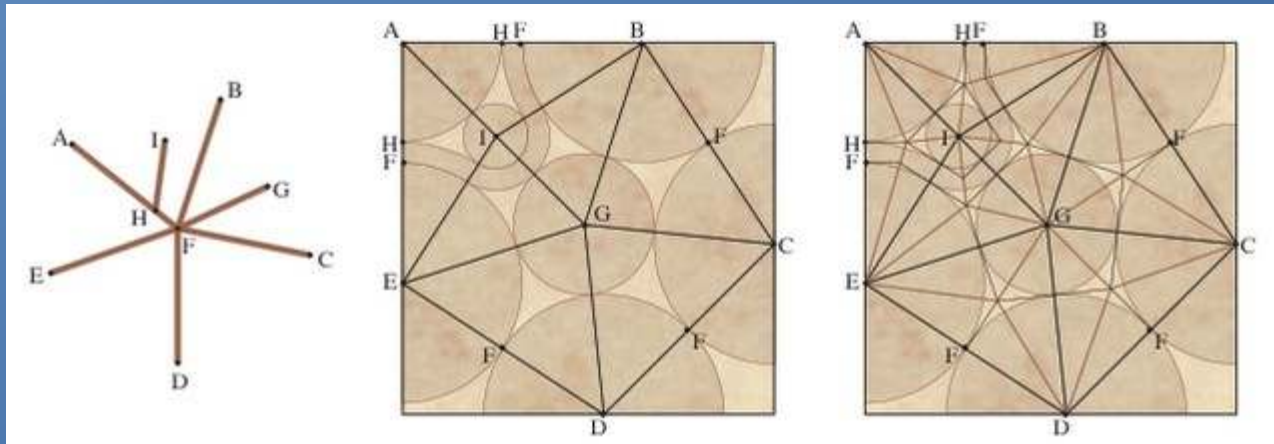
Circle Packing

- Understanding crease patterns using circles

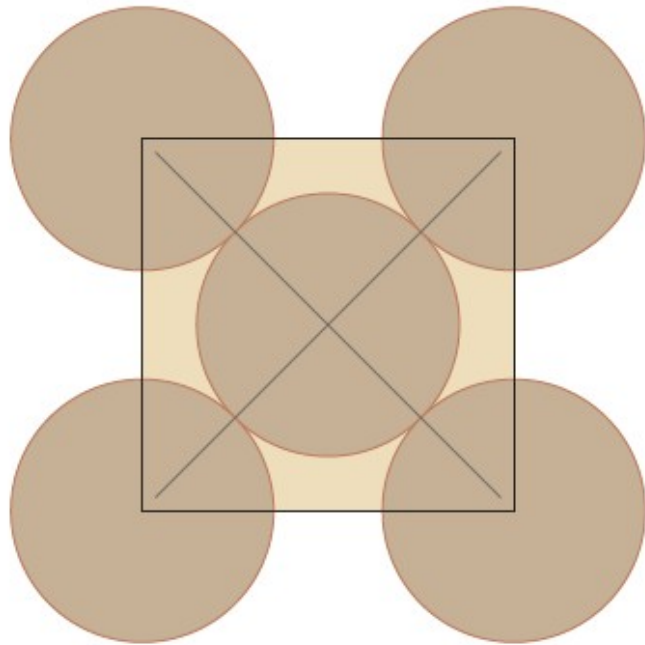


Circle Packing

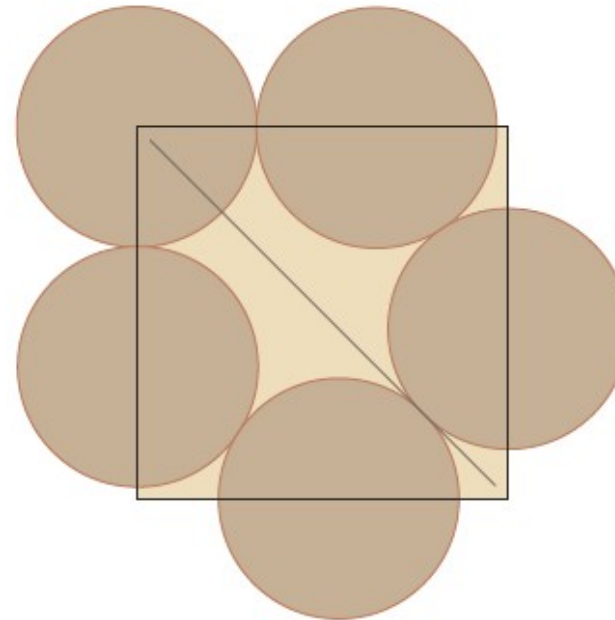
- Design algorithm
 - Uniaxial tree theory
 - Universal molecule



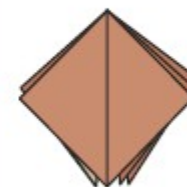
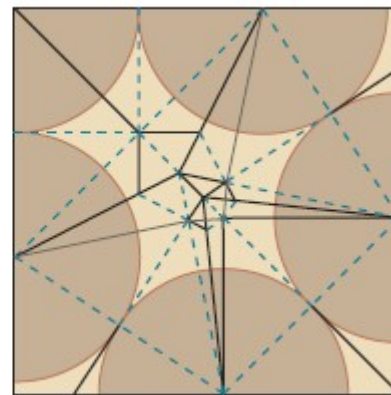
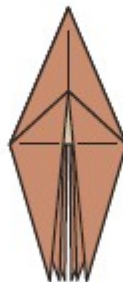
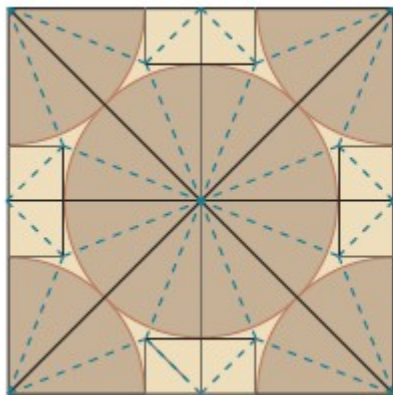
Circle Packing



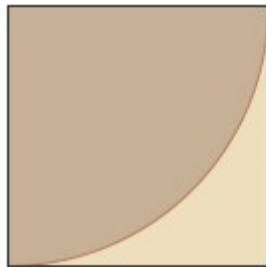
$r = 0.354$



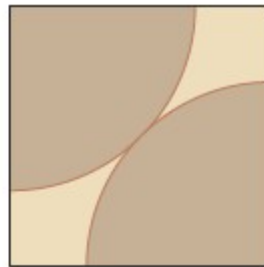
$r = 0.324$



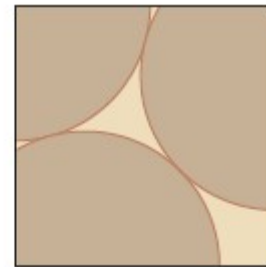
Circle Packing



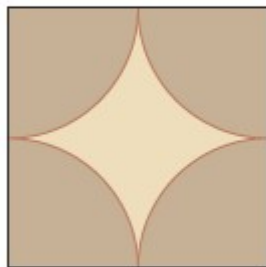
$N = 1$
 $r = 1.000$



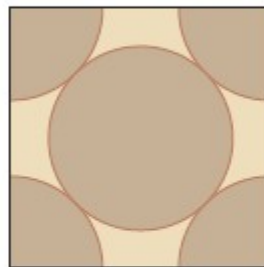
$N = 2$
 $r = 0.707$



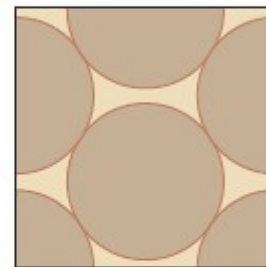
$N = 3$
 $r = 0.518$



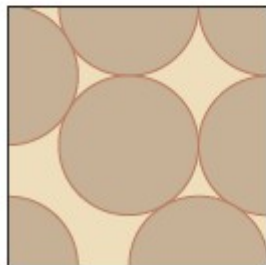
$N = 4$
 $r = 0.500$



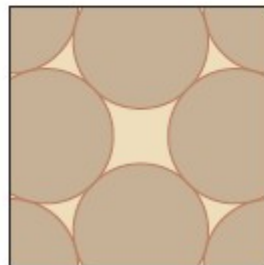
$N = 5$
 $r = 0.354$



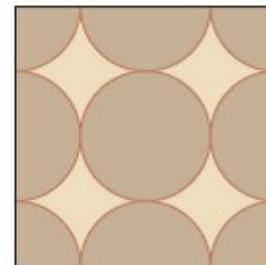
$N = 6$
 $r = 0.300$



$N = 7$
 $r = 0.270$

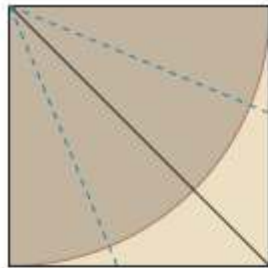


$N = 8$
 $r = 0.259$

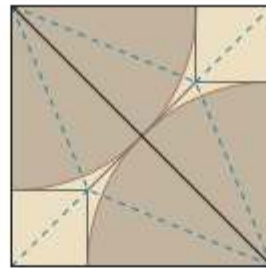


$N = 9$
 $r = 0.250$

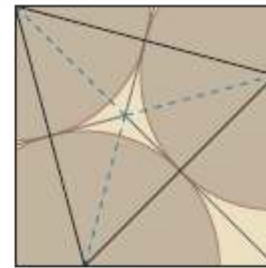
Circle Packing



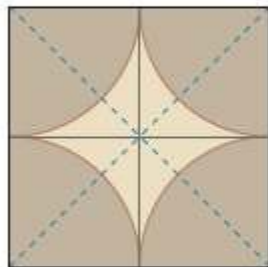
$N=1$



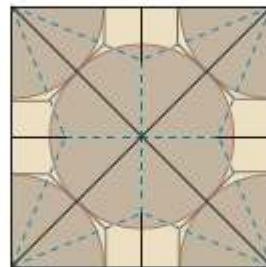
$N=2$



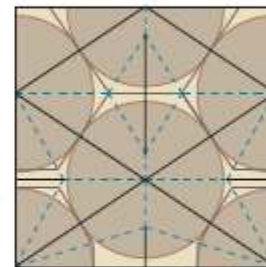
$N=3$



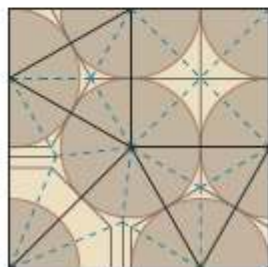
$N=4$



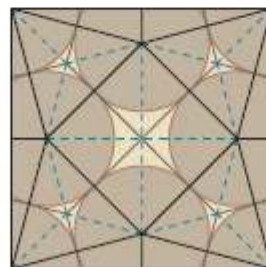
$N=5$



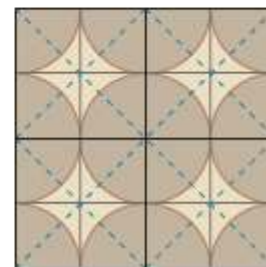
$N=6$



$N=7$

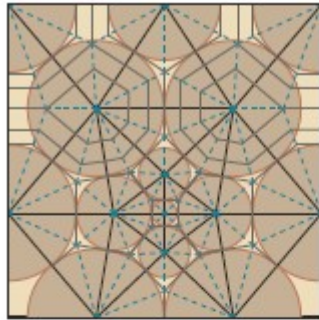


$N=8$

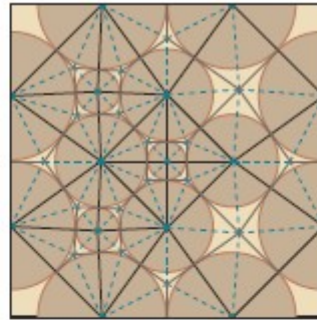


$N=9$

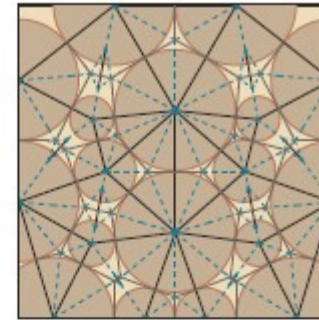
Circle Packing



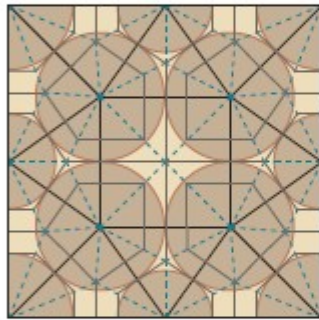
(a)



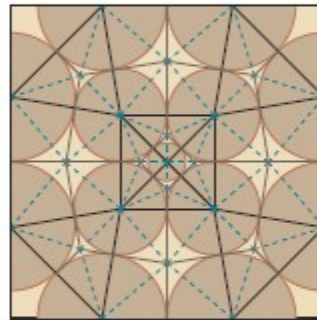
(b)



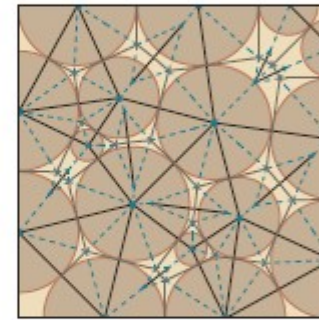
(c)



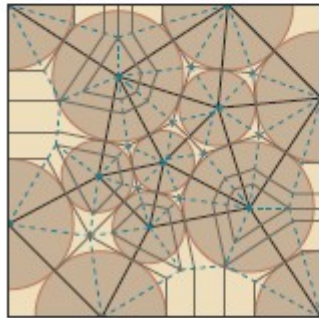
(d)



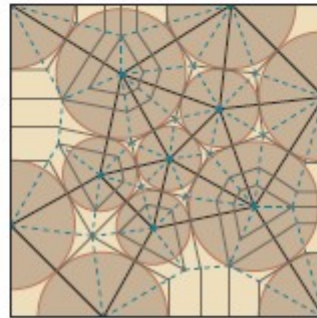
(e)



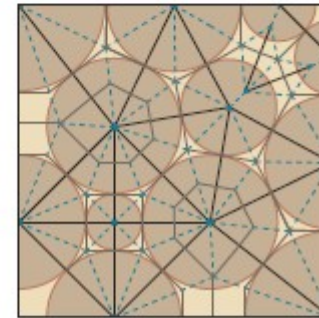
(f)



(g)

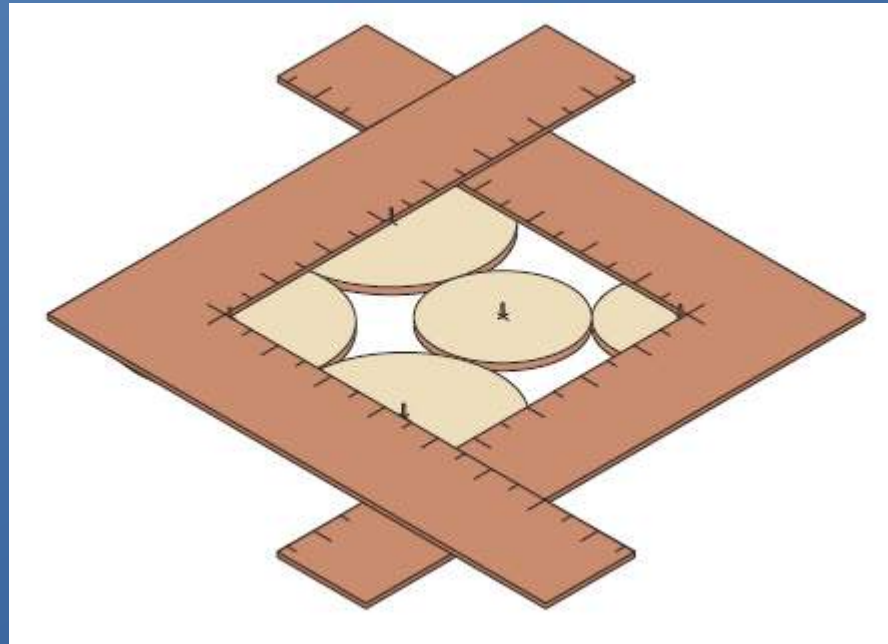
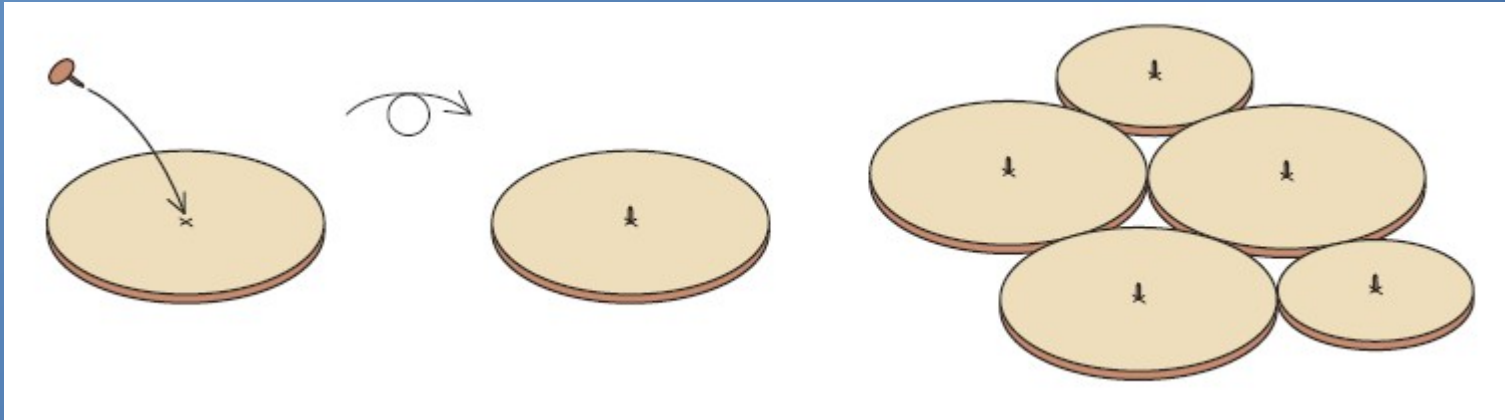


(h)



(i)

Circle Packing

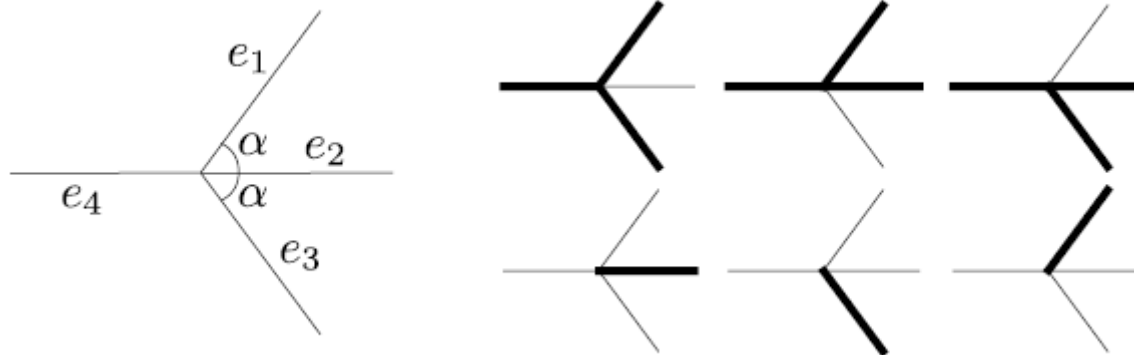
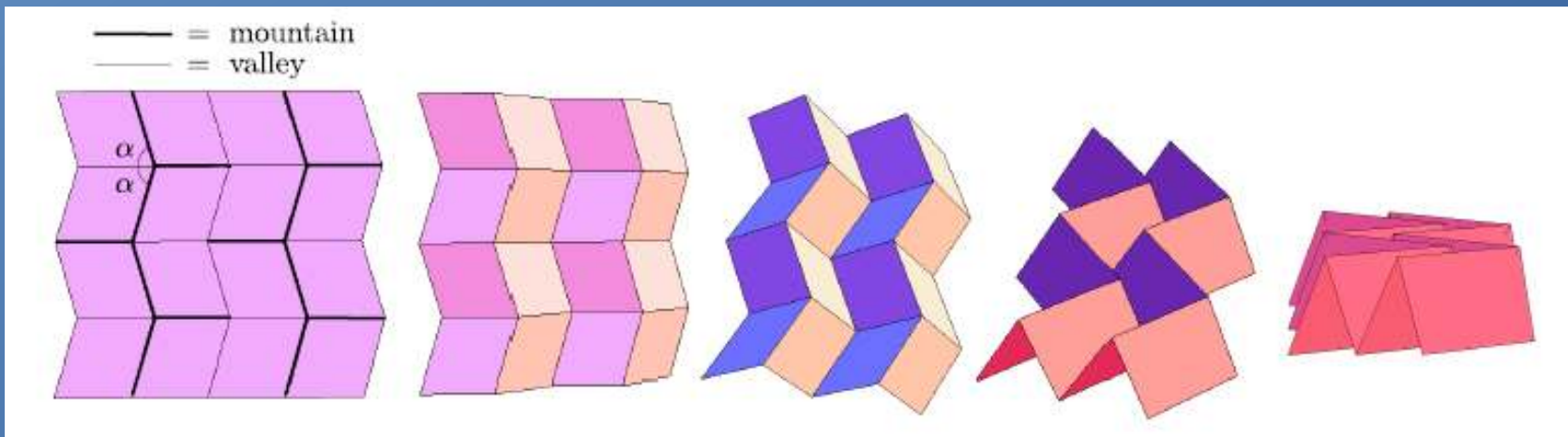


Circle Packing

- Software TreeMaker automates solving the circle packing problem
- Non-linear constrained optimization problem

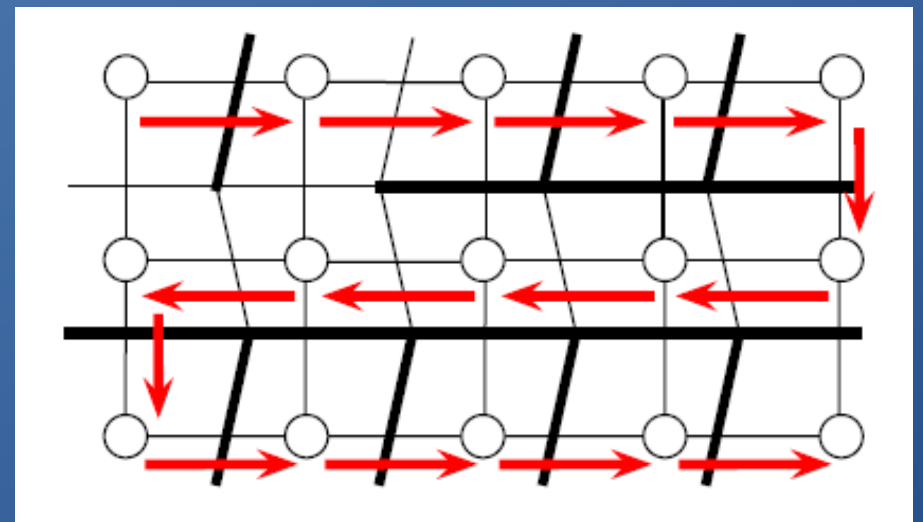
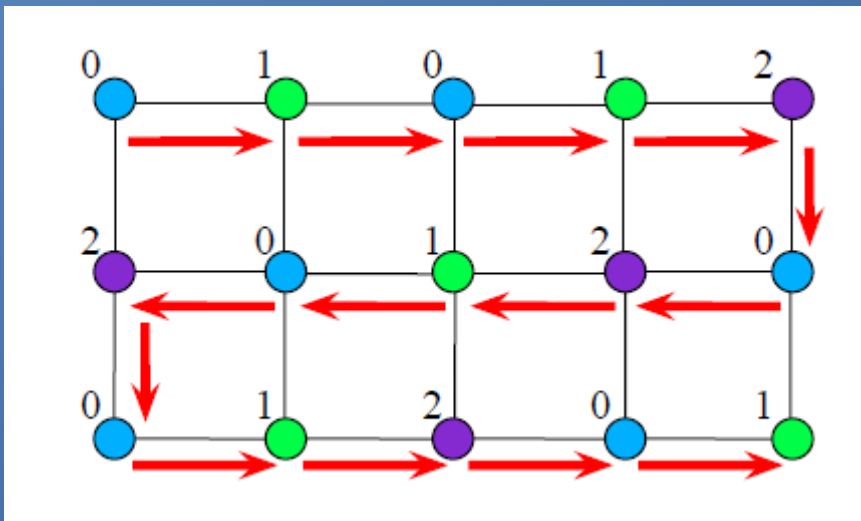
Coloring Problems

- Miura-ori: row staggered pattern
- One angle parameter



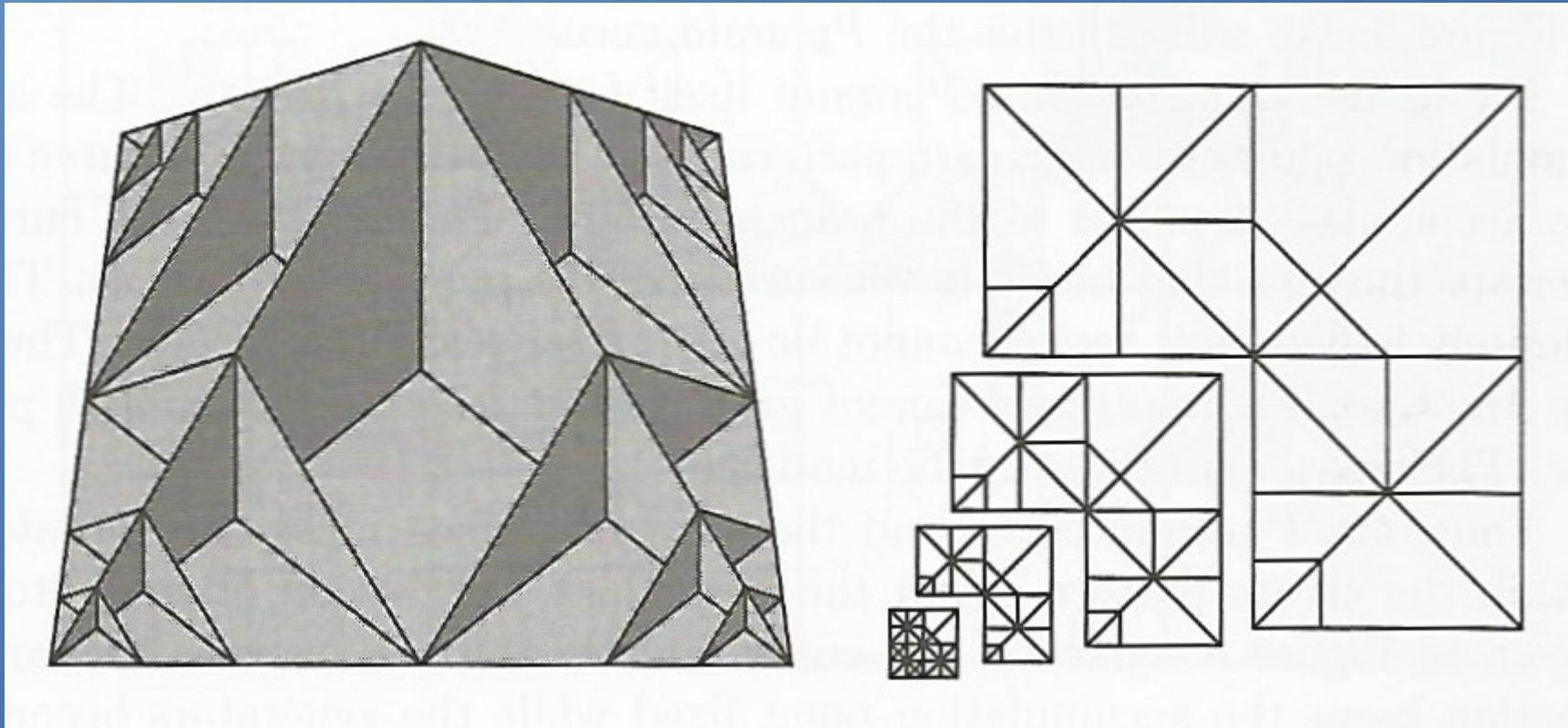
Coloring Problems

- Miura-ori: 3-colorings of the square lattice
- Equivalent to an ice problem in statistical mechanics
- Asymptotic number of colorings is $(4/3)^{3N/2}$

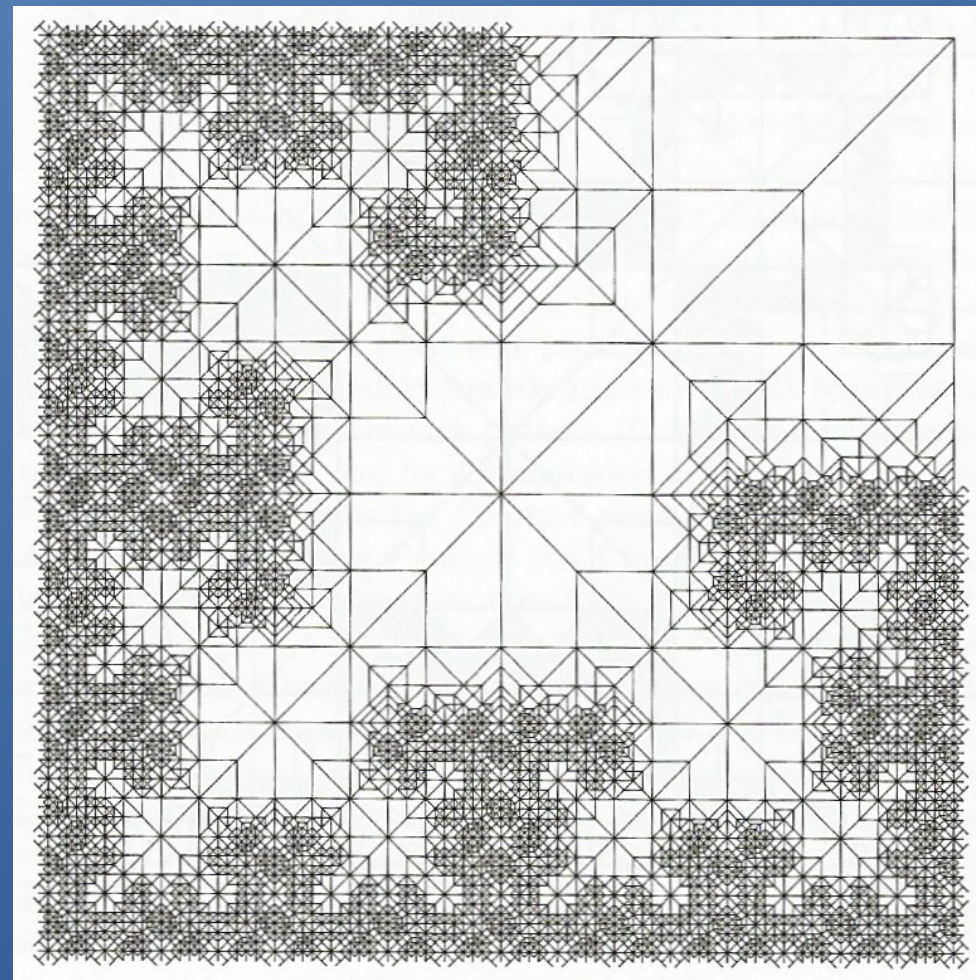
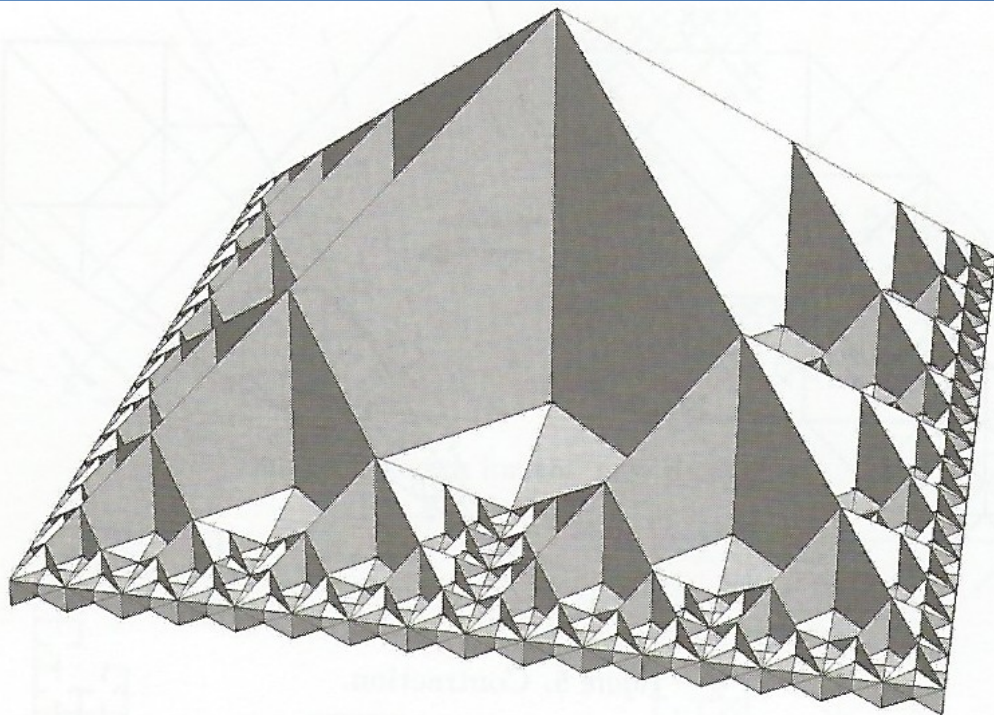


Beyond Flat 2D origami

Fractal Origami



Fractal Origami



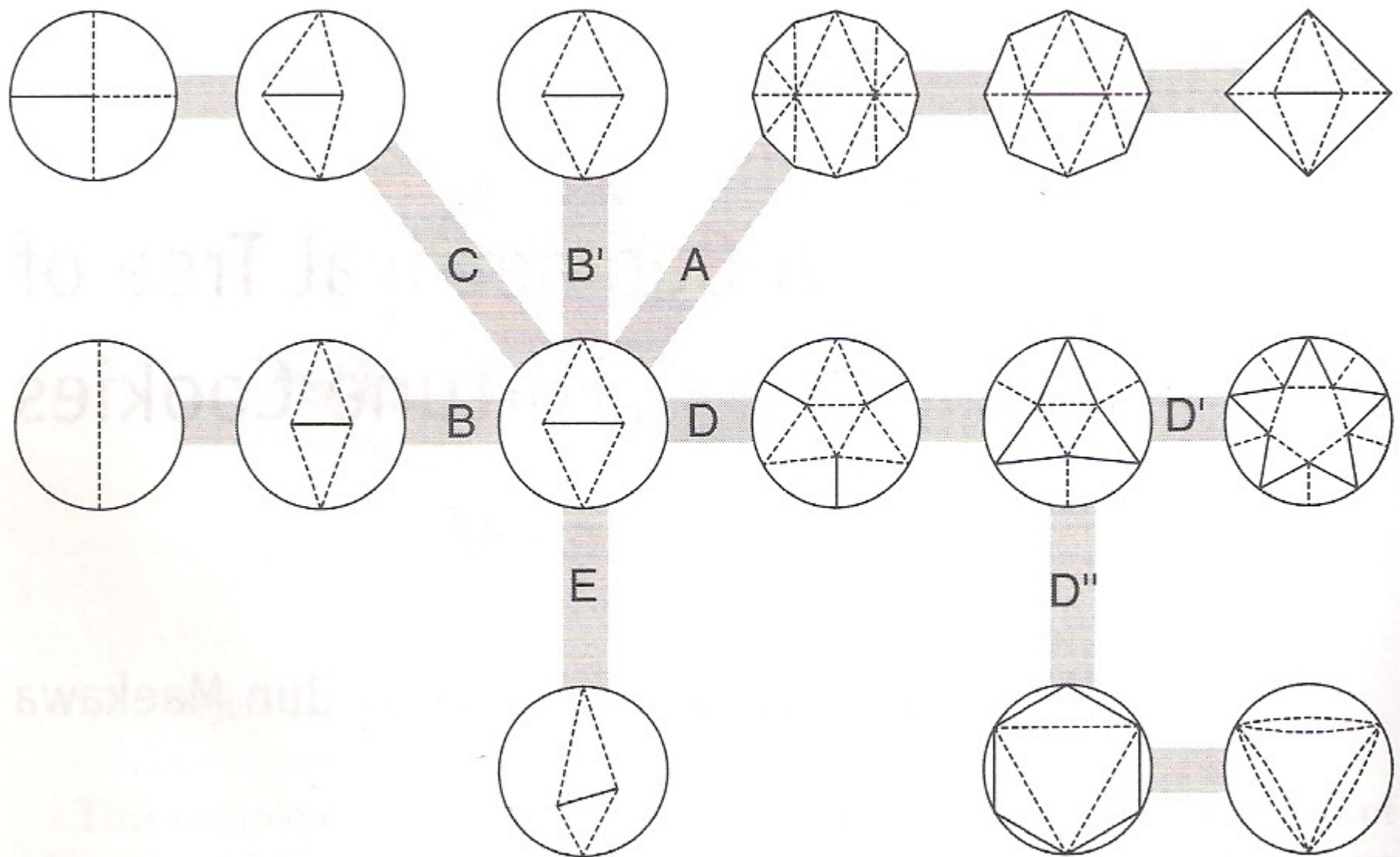
Fractal Origami



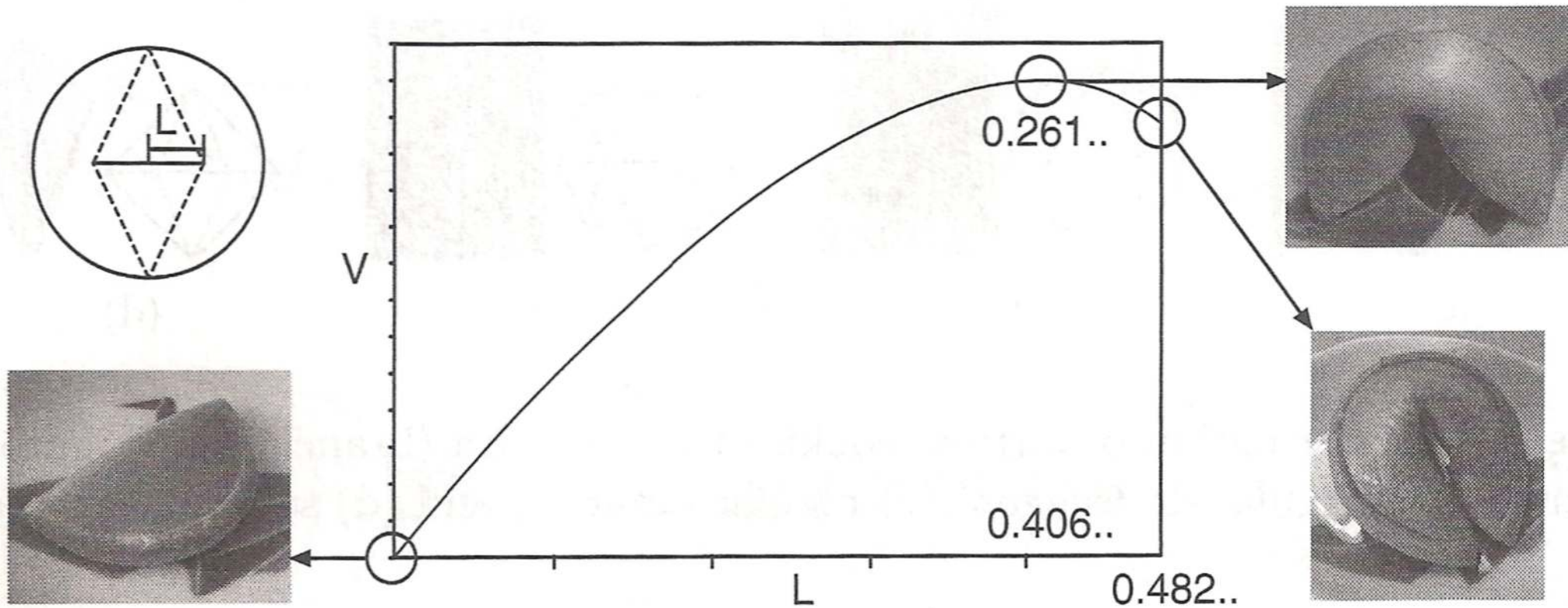
Breaking Flat Foldability



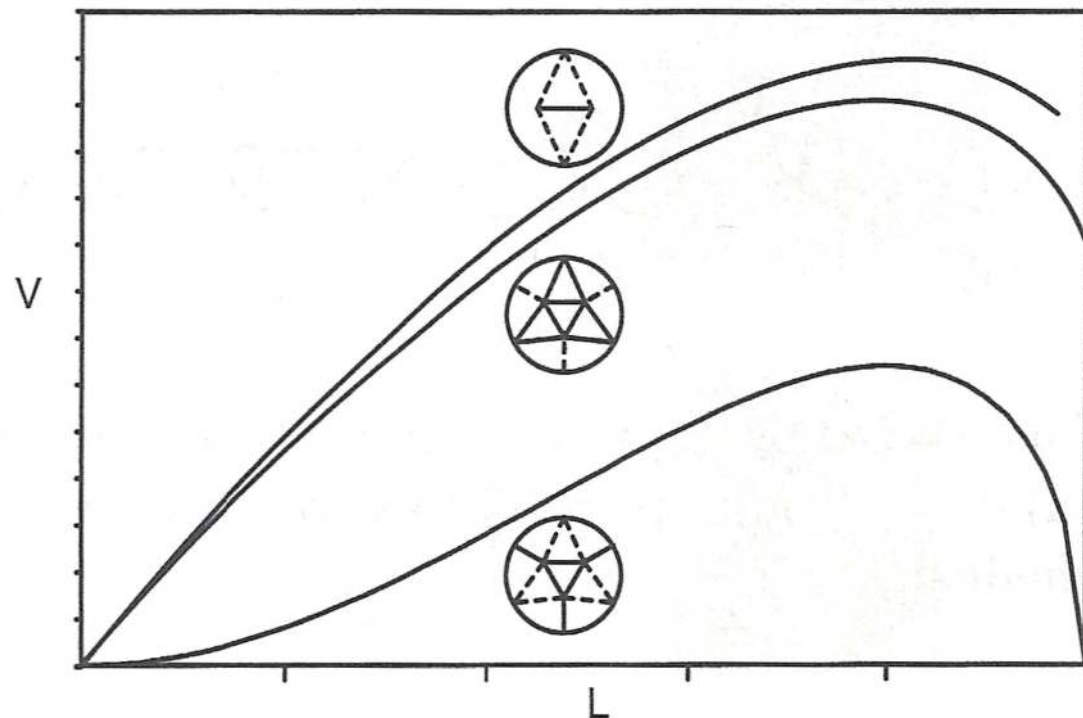
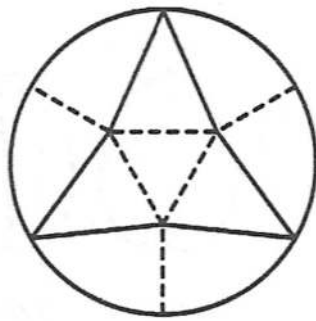
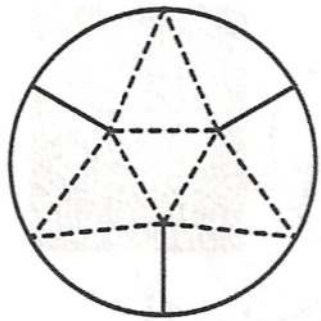
Breaking Flat Foldability



Breaking Flat Foldability



Breaking Flat Foldability



Breaking Flat Foldability

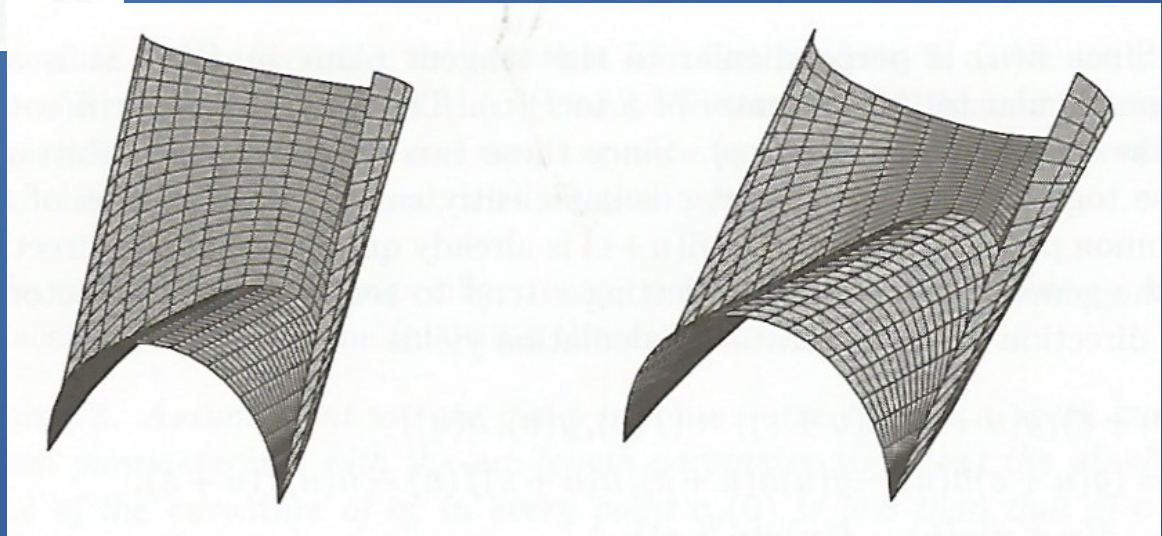
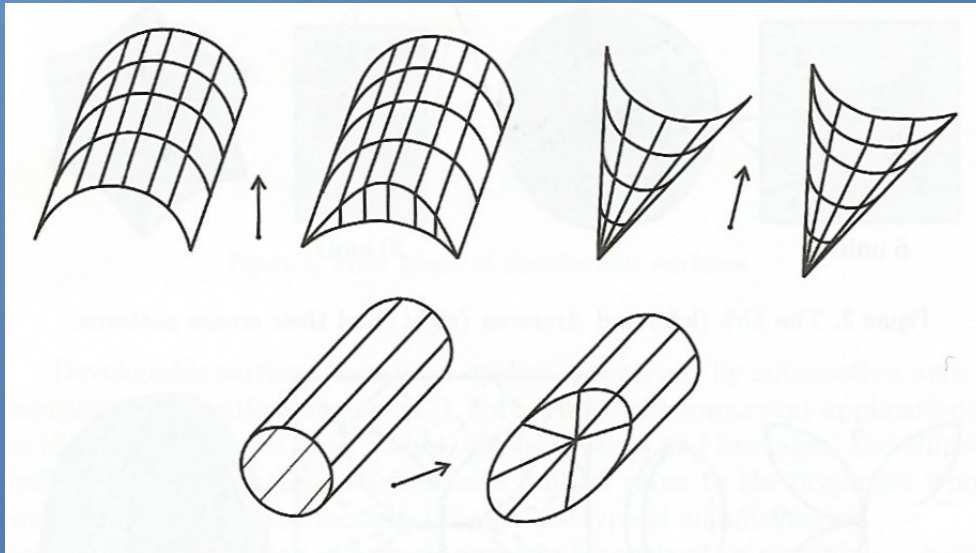
- Explorations:
 - Surface Area
 - Volume
 - Optimization problem
 - Other shapes

Non-flat paper



Non-flat paper

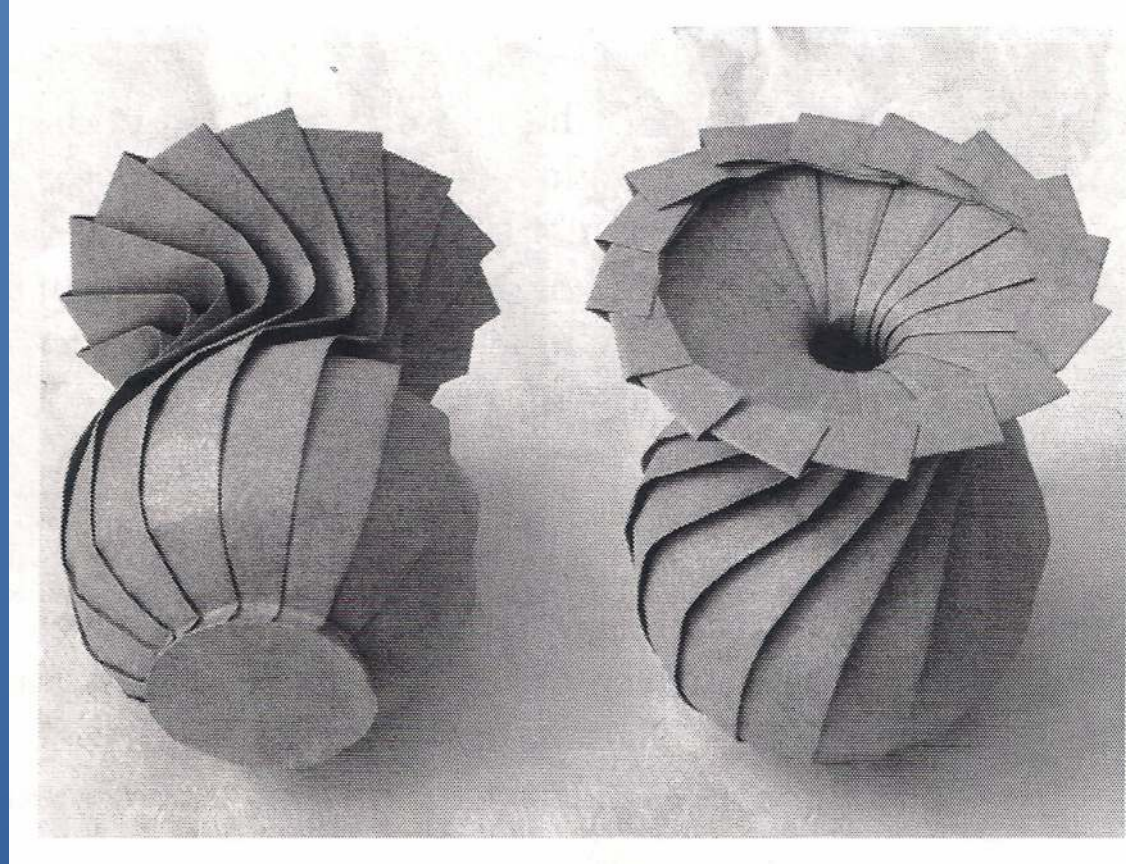
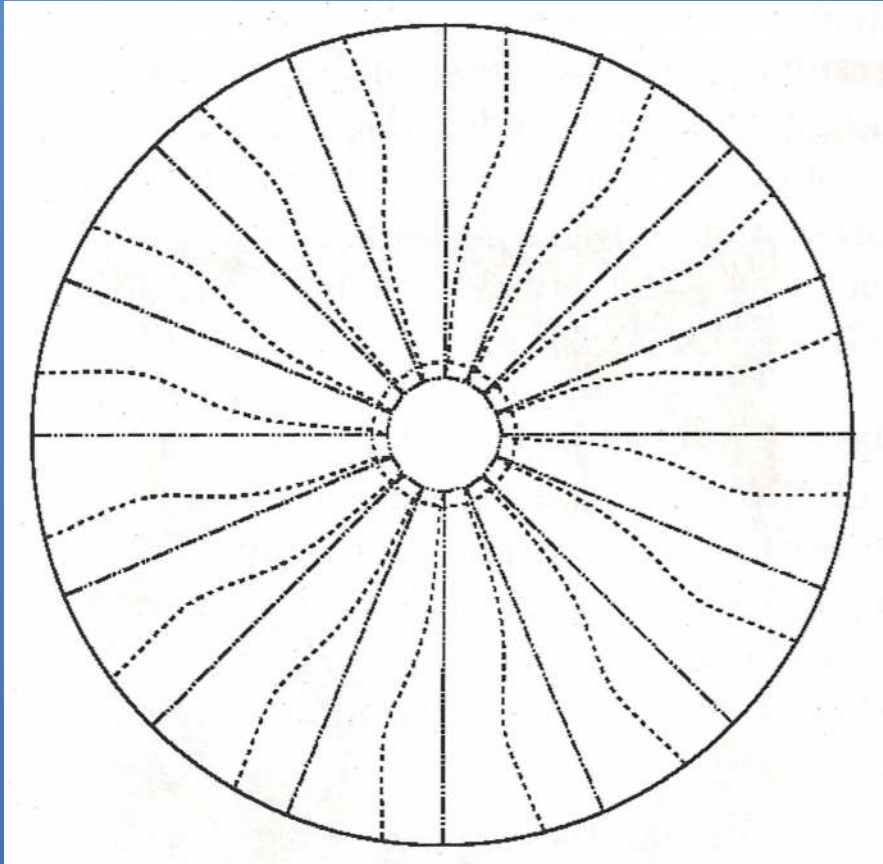
- Conics



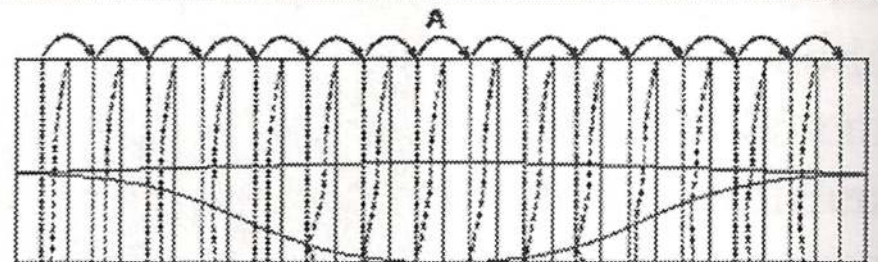
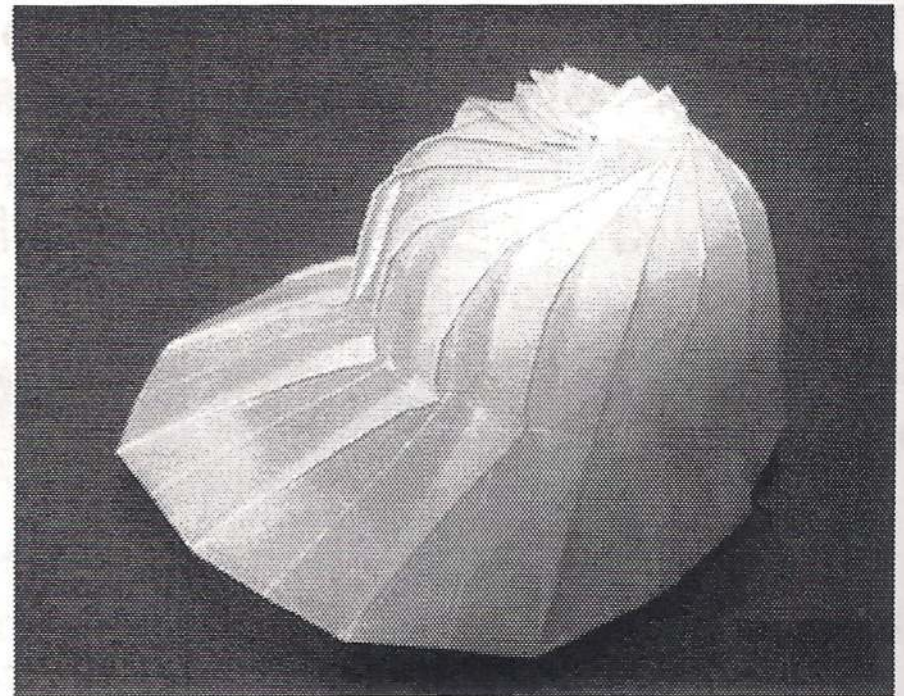
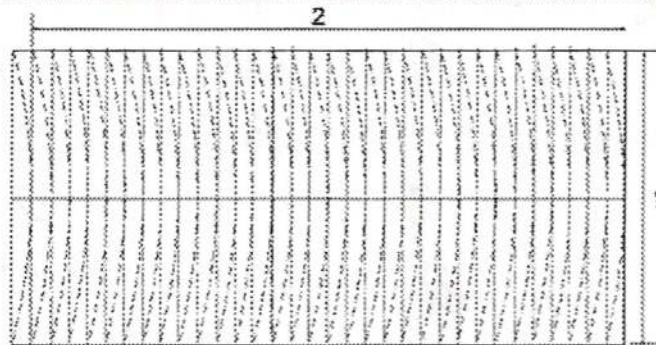
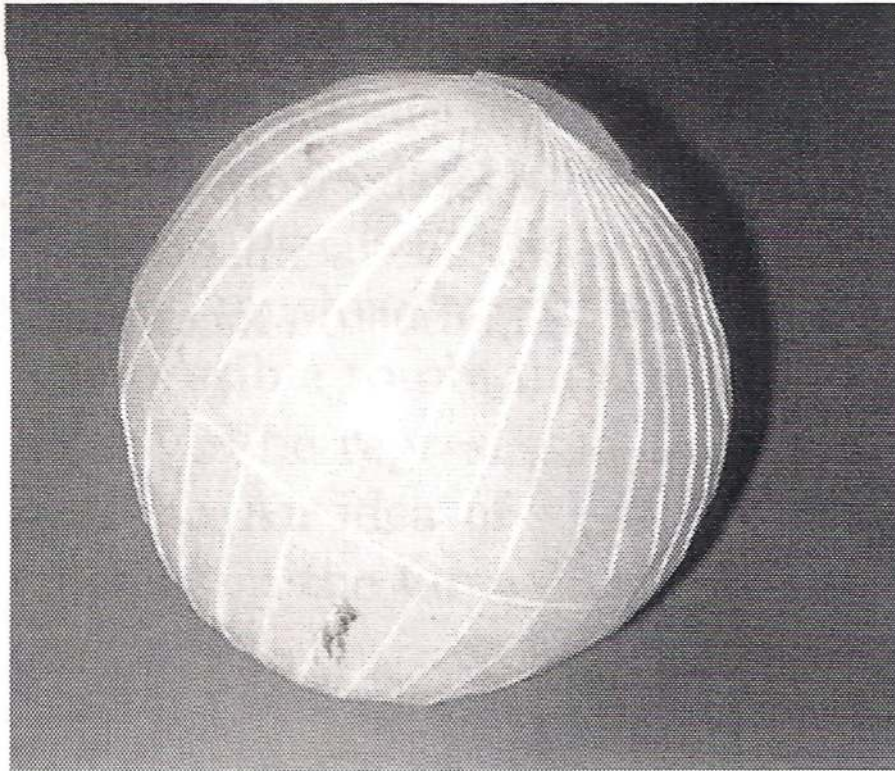
Non-flat paper

- Spherical paper, hyperbolic paper
 - One fold constructions are known

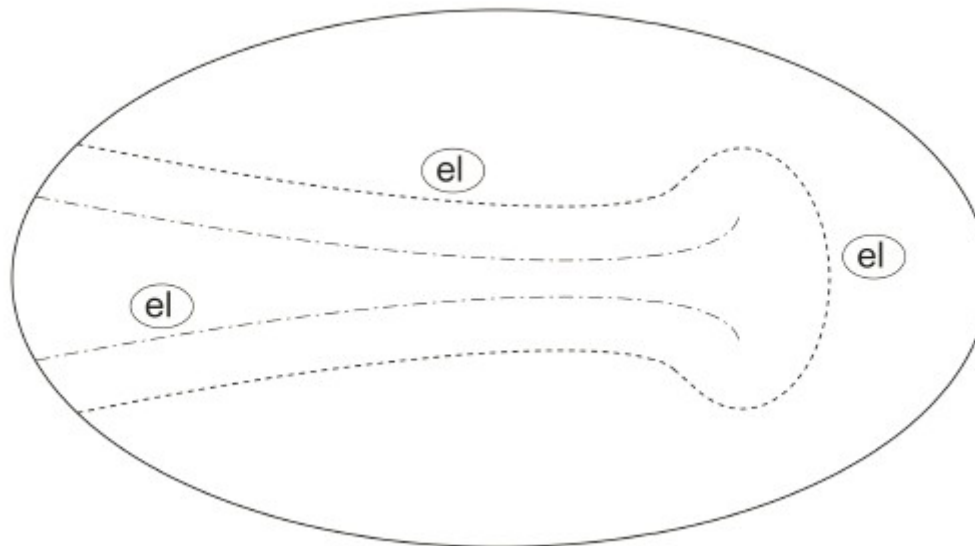
Curved Folding



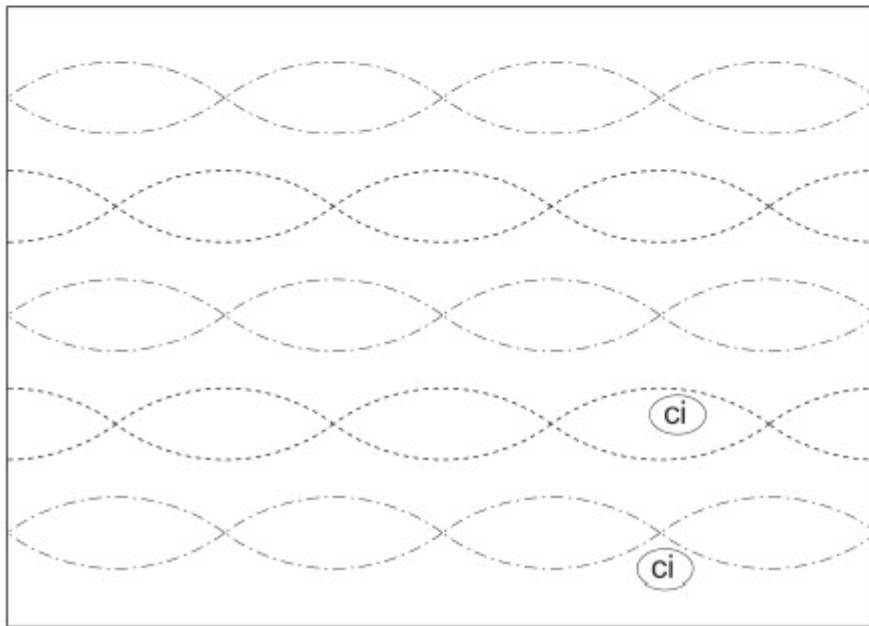
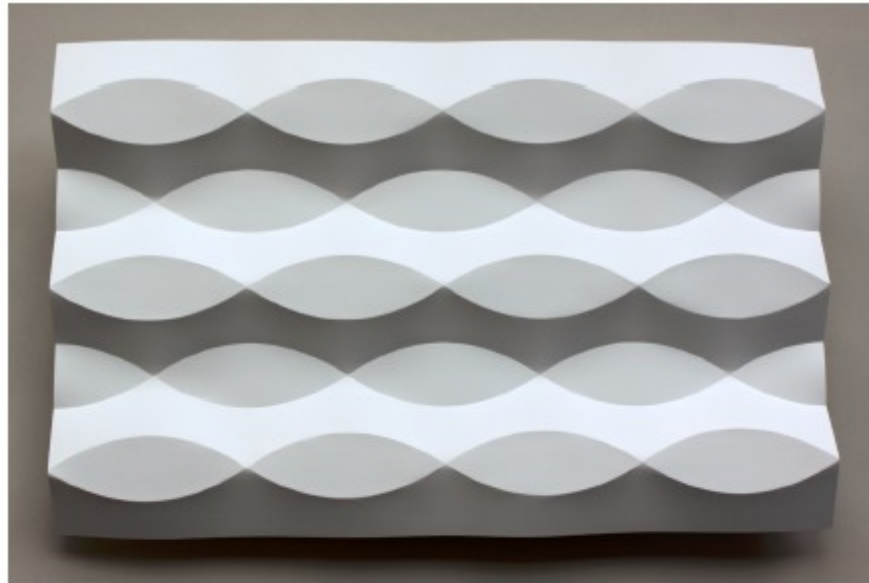
Curved Folding



Curved Folding



Curved Folding



Curved Folding



Curved Folding

- No systematic algorithm for design known
- Direct applications in differential geometry
- Curved folding on non-flat paper not yet explored

A World Of Origami Maths

- Areas of mathematics involved only limited by imagination
- Many more applications in textbooks and convention proceedings
- Many simple research projects are awaiting students and teachers

Thank You!

